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Too Odd (Not) to Be True? A Reply to Olsson

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ABSTRACT

In 'Corroborating Testimony, Probability and Surprise', Erik J. Olsson ascribes to L. Jonathan Cohen the claims that if two witnesses provide us with the same information, then the less probable the information is, the more confident we may be that the information is true (C), and the stronger the information is corroborated (C*). We question whether Cohen intends anything like claims (C) and (C*). Furthermore, he discusses the concurrence of witness reports within a context of *independent* witnesses, whereas the witnesses in Olsson's model are not independent in the standard sense. We argue that there is much more than, in Olsson's words, 'a grain of truth' to claim (C), both on his own characterization as well as on Cohen's characterization of the witnesses. We present an analysis for independent witnesses in the contexts of decision-making under risk and decision-making under uncertainty and generalize the model for n witnesses. As to claim (C*), Olsson's argument is contingent on the choice of a particular measure of corroboration and is not robust in the face of alternative measures. Finally, we delimit the set of cases to which Olsson's model is applicable.

In 'Corroborating Testimony, Probability and Surprise', Erik J. Olsson ([2002]) takes L. Jonathan Cohen to be making the following claims in *The Probable and the Provable* ([1977], p. 98):

(C) The smaller the prior probability of the proposition that two court witnesses agree upon, the greater our degree of confidence will be that their testimony is true.

(C*) The smaller the prior probability of the proposition that two court witnesses agree upon, the stronger it is corroborated[1] by their testimony.

[1] It would be more fitting for Cohen and Olsson to talk about the strength of *confirmation*, since 'corroboration' carries with it the vestiges of Popper's program which turns its back on inductive approaches to investigate the relationship between hypothesis and evidence.

Olsson constructs a clever and elegant model to assess under what conditions these claims are true. The upshot of his argument is that there is only 'a grain of truth' in claim (C): the claim is not 'correct as it stands [...], not even under a charitable rendering'. Claim (C*) purportedly holds only if we make certain special assumptions about our epistemic status with respect to the reliability of the witnesses.

The first question that concerns us is whether Cohen really intends to be making claims like (C) and (C*). Let us look at the pages preceding the ones from which Olsson draws his quotes. Cohen is trying to give an account of the fact that concurring reports (i.e. reports with the same content) from independent witnesses corroborate what is being reported to a greater extent than a single report does. He discusses Boole's account of this fact, which originated in Jakob Bernoulli's *Ars conjectandi*. Either the two witnesses whose reports concur are truth-tellers or they are liars. Let the chance that they are truth-tellers be p and q for the respective witnesses. If the witnesses have a choice between reporting one of two options, then their reports will concur if and only if they are both telling the truth or they are both lying. Hence, the chance that they are both truth-tellers given that their reports concur is:

$$
\begin{aligned}
P(\text{Truth-Tellers}|\text{Concurrence}) &= \frac{P(\text{Truth-Tellers \& Concurrence})}{P(\text{Concurrence})} \\
&= \frac{P(\text{Truth-Tellers})}{P(\text{Truth-Tellers or Liars})} \\
&= \frac{pq}{pq + (1-p)(1-q)}
\end{aligned}
$$

Notice, however, that the chance that we are dealing with truth-tellers goes up as the number of reports increases from one to two if and only if p, $q > 0.50$ on Boole's formula. Cohen objects to this result. He envisions two independent witnesses who are rather unreliable, i.e. p, $q < 0.50$. If their reports are concurring, then 'Boole's formula produces a lower probability for their joint veracity, whereas normal juries would assign a higher one' (*ibid.*, p. 96). Indeed, if $p = q = 0.20$, then it is easy to verify that the chance that two witness reports rather than one witness report is true goes down from 0.20 to approximately 0.06 on Boole's formula. The reason that the formula fails is that there are not just two options in a typical case of witness reports in court, but 'so many different things can be said instead of the truth' (*ibid.*, p. 97). Hence, the chance that two liars will produce concurring reports is no longer $(1-p)(1-q)$, but much lower.

It is in this context that we should assess the claim of Cohen quoted by Olsson: 'Where agreement is relatively improbable (because so many different things might be said), what is agreed is more probably true' (*ibid.*, p. 98). The question is: more probably true than when? More probably true than when the agreement had been less improbable (because there were fewer things to say)? This is Olsson's way of filling in the ellipsis. Or more probably true than when there had been only one witness report? This is the comparison that Cohen intended. Cohen is arguing that concurring reports yield a higher degree of confidence that what is reported is true than does a single report. This is so even when the chance that the witnesses are reliable is relatively low. The reason that Boole's formula fails in this context is that it is typically not the case in court proceedings that 'the domain of possibilities is a binary one' (*ibid*., p. 97). Cohen then goes on to develop his own account and lays out a set of sufficient conditions for concurring reports to yield a higher degree of confidence than a single report.

But we are not sticklers for textual interpretation. Although it is not what Cohen had in mind, claims (C) and (C*) do seem to square with a certain intuition and it is worth investigating whether we can give a probabilistic account of this intuition.

There is a second caveat. Olsson chooses to model claims (C) and (C*) for witnesses who are both either fully reliable or fully unreliable. But Cohen is discussing court witnesses who are *independent* whereas the witnesses in Olsson's set-up are not independent in the standard sense of the term. Nonetheless, it is interesting to see whether there is indeed no more than a grain of truth to claim (C) for witnesses who are either both reliable or both unreliable.

In Section 1, we defend claim (C) against Olsson's charge while adopting his own probabilistic characterization of the relationship between the witnesses. In Section 2, we defend claim (C) against Olsson's charge while respecting the stipulation that the witnesses are independent. In Section 3, we focus on claim (C*). Olsson's argument points to an interesting feature in confirmation theory. A wide range of measures of corroboration (or measures of confirmation, support, relevance, weight-of-evidence, ...) have been developed and defended in the literature. These measures behave very differently as a function of the prior probability of the hypothesis. Olsson argues against claim (C*) by focusing on one such measure. We will show that his conclusion is not *robust* in the face of alternative measures. In Section 4, we sketch some historical background of 'too odd not to be true' reasoning and argue that Olsson's model only applies to a restricted set of cases.

1 Claim (C) examined for Olsson's characterization of the relationship between the witnesses

Cohen discusses concurring reports for propositions that are relatively improbable. It is fair to say that a claim is relatively improbable only if—at the very least—it is less probable than not. This is also in keeping with

Cohen's examples. He points to examples where there are multiple equally plausible suspects for a crime. In Section 3, Olsson shows that claim (C) does not hold when we know the degree of reliability of the witnesses. This is fair enough and entirely in the spirit of the context that Cohen is discussing. In medicine, we often have knowledge of the percentage of false positives and false negatives (and hence of the reliability) of medical tests that 'bear witness' of some disease. However, we do not have any such knowledge at our disposal in a court situation. Olsson shows that a substantially weaker version of (C) holds true when the reliability of the witnesses is unknown. This he takes to be the grain of truth in claim (C). We will argue that, his results notwithstanding, there is significantly more than just a grain.

Cohen's quotes concerning concurring reports occur within the context of two *independent* witnesses.[2] Smith and Jones are independent witnesses in the standard sense of the term[3] if and only if the following condition holds (Bovens and Hartmann [2002], p. 37; Earman [2000], p. 57; Fitelson [2001a], pp. 127–30):

$$P(\mathrm{E}_i|\mathrm{H}, \mathrm{E}_j) = P(\mathrm{E}_i|\mathrm{H}) \text{ for } i, j = 1, 2 \text{ and } i \neq j \tag{1}$$

$$P(\mathrm{E}_i|\neg\mathrm{H}, \mathrm{E}_j) = P(\mathrm{E}_i|\neg\mathrm{H}) \text{ for } i, j = 1, 2 \text{ and } i \neq j \tag{2}$$

Olsson examines claim (C) for witnesses who do not satisfy these independence conditions. There are two crucial features to Olsson's set-up.

[2] Cohen goes on to show that a set of conditions that are weaker than independence is sufficient for concurring reports to yield a higher degree of confidence than a single report ([1977], pp. 101–7). It is easy to show that Olsson's set-up does not satisfy these conditions either. However Cohen's conditions are not necessary conditions and it can be shown that concurring reports will also yield a higher degree of confidence than a single report in Olsson's set-up.

[3] Cohen also subscribes to this conception of independent witnesses. First, he argues that the reports of the witnesses can only be causally connected through the truth of what is testified: there should be 'no special reason', i.e. other than the truth of what is testified, 'to suppose that the two testimonies would be identical' ([1977], p. 94). Hence, if we know the truth-value of what is testified, i.e. the truth-value of H, then learning that one witness testified that H should not affect the chance that the other witness testified that H. Second, when Cohen lays out sufficient conditions for corroboration, his conditions (4) and (5) ([1977], p. 104) are entailed by our conditions (1) and (2), respectively: Cohen argues that we do not need strict independence, but there may also be negative relevance between the reports, given that H is false, and there may also positive relevance between the reports, given that H is true, for corroboration to occur. Finally, for readers who are familiar with the graphical representation of conditional independence structures in Bayesian Networks, the structure of Olsson's set-up for unknown reliability can be represented by a Bayesian Network with four nodes, one for the hypothesis, one for the reliability of the witnesses, and a node for the report of each witness. There are arrows from the hypothesis and from the reliability to the witness reports. The causal link between the reports is not only through the hypothesis as a confounder, but also through the reliability as a confounder: hence, the causal link between the reports is not only through 'the truth of what is testified'. Independence requires that there is no other causal link than through the hypothesis as a confounder. In our set-up for independent witnesses, we have five nodes—one for the hypothesis, one for the report of each witness, and one for the reliability of each witness. There are arrows from the hypothesis to the reports and from the reliability of the witnesses to their respective reports. The only causal link between the reports is through the hypothesis as a confounder.

First, either both witnesses are reliable (R) or both witnesses are not reliable (U). Hence, $P(\mathrm{U}) = 1 - P(\mathrm{R})$. Second, if they are reliable, they are truth-tellers, and if they are not reliable, then the chance that they will incriminate Forbes is equal to the prior probability that Forbes committed the crime. The assumptions of the model are captured in premises (i) through (viii) in Section 5 in Olsson: for $i, j = 1, 2$ and $i \neq j$, (i) $P(\mathrm{E}_i|\mathrm{H, R}) = 1$; (ii) $P(\mathrm{E}_i|\neg\mathrm{H, R}) = 0$; (iii) $P(\mathrm{E}_i|\mathrm{H, U}) = P(\mathrm{H})$; (iv) $P(\mathrm{E}_i|\neg\mathrm{H, U}) = P(\mathrm{H})$; (v) $P(\mathrm{E}_i|\mathrm{H, U, E}_j) = P(\mathrm{E}_i|\mathrm{H, U})$; (vi) $P(\mathrm{E}_i|\neg\mathrm{H, U, E}_j) = P(\mathrm{E}_i|\neg\mathrm{H, U})$; (vii) $P(\mathrm{R}|\mathrm{H}) = P(\mathrm{R}) = P(\mathrm{R}|\neg\mathrm{H})$; (viii) $P(\mathrm{U}|\mathrm{H}) = P(\mathrm{U}) = P(\mathrm{U}|\neg\mathrm{H})$.[4] To see that Olsson's witnesses are not independent, consider the following two theorems. (All theorems are proven in the Appendix.)

Theorem 1 If (i) through (viii), then $P(\mathrm{E}_i|\mathrm{H, E}_j) > P(\mathrm{E}_i|\mathrm{H})$ for $i, j = 1, 2$ and $i \neq j$.

Theorem 2 If (i) through (viii), then $P(\mathrm{E}_i|\neg\mathrm{H, E}_j) > P(\mathrm{E}_i|\neg\mathrm{H})$ for $i, j = 1, 2$ and $i \neq j$.

Obviously theorems 1 and 2 conflict with (1) and (2). However, Olsson's set-up is not without interest: we may well wish to know whether claim (C) holds up when both witnesses are either fully reliable or fully unreliable. Let the prior probability that the witnesses are fully reliable be $P(\mathrm{R}) = r$; the prior probability that Forbes committed the crime be $P(\mathrm{H}) = h$, and the posterior probability that Forbes committed the crime after having learned from both witnesses that he committed the crime be $P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = h^*$. To assess Olsson's claim that there is only a grain of truth in claim (C), picture a box in which h is plotted against r. The variable r is contained in the open set (0, 1), while h is contained in the open set (0, 0.5), since we are only interested in relatively improbable propositions. What we want to know is the following. For which pairs (r, h) in this box does claim (C) hold? In other words, for which pairs (r, h) can it be said that if the prior probability h had been lower, then the posterior probability h^* would have been higher, for the particular value of the variable r. If there is only a grain of truth to (C), then this should become evident when we determine the area in the box where (C) is true.

So let us turn to this task. Olsson derives a formula to calculate the posterior probability h^*.[5] We introduce the convention that $\bar{x}$ stands for $(1 - x)$ for any probability value x. Olsson's formula (U6) can be written as

[4] Note that the conditional probabilities are defined only if we assume that the probability distribution over the random variables H (whose values are H and ¬H) and R (whose values are R and ¬R) is non-extreme. We adopt this assumption throughout the paper.

[5] Compare Theorem 1 in Bovens and Hartmann ([2002]) within the context of confirmation with multiple unreliable instruments. The expression in this theorem contains three more variables, but if the hypothesis is tested directly, then $p = 1$ and $q = 0$, and if an unreliable instrument randomizes at $a = h$, then one obtain the same results.

$$h^* = \frac{r + h^2\bar{r}}{r + h\bar{r}}. \tag{3}$$

Olsson's Figure 2 shows that for $r = 0.10$, there is only a very small range of values of h for which the posterior probability h^* is a decreasing function of h. The grain of truth that he recognizes in claim (C) is that for any value of r, there is a range of values in which the posterior probability h^* is indeed a decreasing function of h.

But is this just a grain of truth? Indeed, what Olsson's example shows is that there are some cases in which claim (C) holds true. But to assess the breadth of this claim, we need to turn his analysis one small notch further. In Olsson's Figure 2, we see that the posterior probability h^* has a minimum for some value of h, when $r = 0.10$. To the left of this minimum the claim is true, and to the right of this minimum the claim is false. So we need to find the minimum for each value of $r \in (0, 1)$. How do we find the minimum in Olsson's Figure 2? The standard technique to do so is to take the partial derivative of h^* with respect to h,

$$\frac{\partial h^*}{\partial h} = \bar{r}\frac{h^2 - \bar{h}^2 r}{(r + h\bar{r})^2}, \tag{4}$$

and to set this partial derivative to 0 and solve for h. The solution for h, $r \in (0, 1)$ is

$$h_{\min} = \frac{\sqrt{r}}{1 + \sqrt{r}}. \tag{5}$$

For $r = 0.10$, this solution yields $h_{\min} < 0.24$, which is indeed where the curve in the graph in Olsson's Figure 2 reaches its minimum.[6] We plot the function in (5) to determine the minima for any value of $r \in (0, 1)$ in Figure 1. For value pairs of (r, h) underneath this curve, claim (C) turns out to be true. Olsson is correct that we need to impose certain constraints on this claim, but it seems to us that given the relative size of the area underneath the curve, there is not just a grain of truth, but rather a heap of truth to claim (C).

Furthermore, we have interpreted relative improbability as being less probable than not. But the case that Cohen has in mind is a case in which the court witnesses identify a suspect 'out of so many possible criminals' (*op. cit.*, p. 98). In this case, we need to consider the area under the curve under the constraint that $h = 1/n$, when there are n equally probable suspects. For

[6] To assure ourselves that this solution yields a minimum rather than a maximum over $r \in (0, 1)$, observe that the second derivative of h^* with respect to h is

$$\frac{\partial^2 h^*}{\partial h^2} = \frac{2r\bar{r}}{(r + h\bar{r})^3}$$

which is positive for $r \in (0, 1)$.

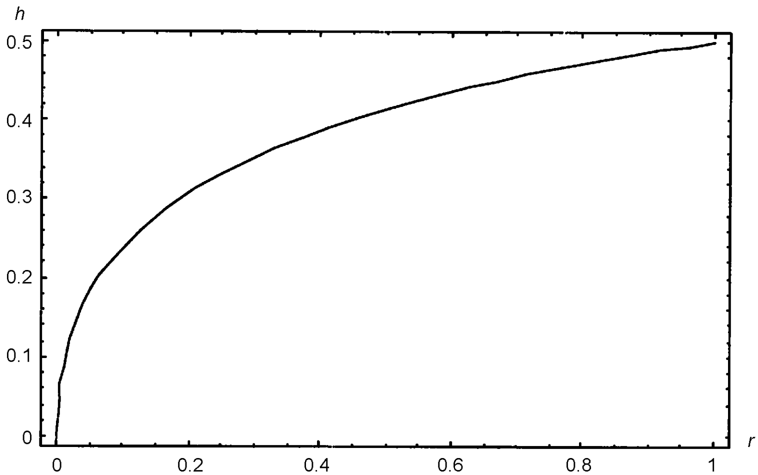

Figure 1. The space under the curve represents all points in which the posterior probability of the hypothesis is a decreasing function of the prior probability of the hypothesis when the witnesses are either both reliable or both unreliable.

example, let $n = 10$. Solving equation (5) for $h = 0.10$, we learn that claim (C) holds true as long as we let $r > 0.012$. In other words, for ten or more suspects, claim (C) is true, as long as our degree of confidence that both of our informers are reliable is greater than ... a mere 1.2%. It seems to us that Olsson has built a model that supports claim (C) under the very plausible assumption that the witnesses are at least minimally reliable, rather than a model that uncovers merely a grain of truth in (C).

One more step in the analysis reveals even more support for claim (C). Unlike for medical tests, it is quite common that we do not have an accurate assessment of the reliability of court witnesses. This is why Olsson conducts his analysis within the framework of *decision-making under risk*: we assign different probabilities to whether the witness is reliable or not. But in reality, we are often not even capable of making *this* assessment. Rather, the situation is one of *decision-making under uncertainty*: we have no clue whatsoever about the chance that the witness is reliable or not. One approach is to invoke the principle of indifference at this point. Let r be a continuous variable whose values range from 0 (for certainty that the witnesses are fully unreliable) to 1 (for certainty that the witnesses are fully reliable). We construct a uniform distribution over r. If we really have no clue how reliable the witnesses are, in the sense that we take any value of r to be equally probable, then for what values of h does claim (C) hold true? We first calculate the expected value of h^* for each value of h, under the supposition that there is a uniform distribution over r:

$$\langle h^* \rangle = \int_0^1 \frac{r + h^2 \bar{r}}{r + h\bar{r}} dr = \bar{h}^{-1}(1 - h^2 + h \ln(h)) \quad \text{for } h \in (0, 1) \tag{6}$$

To calculate the minimum, we take the derivative of this function with respect to h:

$$\frac{d\langle h^* \rangle}{dh} = \bar{h}^{-2}(h^2 - 3h + \ln(h) + 2) \tag{7}$$

set this derivative equal to 0 and solve numerically for $h \in (0, 1)$:

$$h_{\min} \approx 0.32 \tag{8}$$

Hence, if we have no clue whatsoever about the reliability of the witnesses and there are more than three equally probable suspects (so that the prior probability that a particular suspect committed the crime is no greater than 0.25, and thus less than 0.32), then claim (C) holds. The less probable the information is that the witnesses agree upon, the greater our degree of confidence will be that the information is true. Considering that Cohen is interested in a scenario in which there is a wide range of n suspects, it is not an overly charitable read to set the number of suspects at $n > 3$. Furthermore, our judgment about the reliability of a witness in a court is typically a judgment neither under certainty nor under risk, but rather under uncertainty. If we build these plausible assumptions into Olsson's analysis, then the model provides direct validation for claim (C).

2 Claim (C) examined for two or more independent witnesses

What happens when we run Olsson's analysis for witnesses who are genuinely independent, i.e. who satisfy our conditions (1) and (2) in Section 1? Let us follow the same route of analysis for independent witnesses. In the Appendix we have calculated the posterior probability after receiving corroborating reports from two independent witnesses, making similar assumptions as in Olsson's model:[7]

Theorem 3

$$P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = h^* = \frac{(h + \bar{h} r_1)(h + \bar{h} r_2)}{h + \bar{h} r_1 r_2}$$

in which r_i is the probability that witness i is reliable for $i = 1, 2$. From here on we follow the same procedure as in Section 1. We have plotted the curve

[7] Compare Theorem 2 in Bovens and Hartmann ([2002]) within the context of confirmation with multiple unreliable instruments. Making the same substitutions as in note 3, this result can be obtained for the special case $r_1 = r_2 = r$.

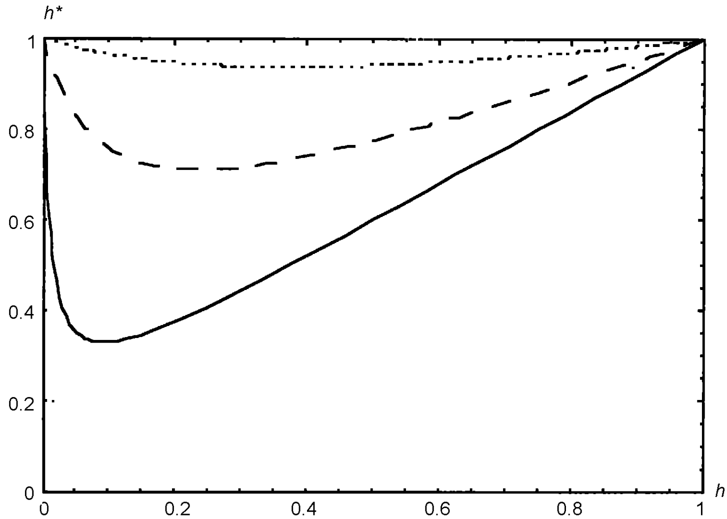

Figure 2. The posterior probability of the hypothesis as a function of the prior probability of the hypothesis when the witnesses are independent and there is an $r_i = 0.10$ (full line), 0.30 (dashed line) and 0.60 (dotted line) chance that a single witness is reliable.

for independent witnesses with the same reliability for $r_1 = r_2 = 0.10$, 0.30, 0.60 in Figure 2.

For decision-making under risk, we calculate the partial derivative with respect to h:

$$\frac{\partial h^*}{\partial h} = \bar{r}_1 \bar{r}_2 \frac{h^2 - \bar{h}^2 r_1 r_2}{(h + \bar{h} r_1 r_2)^2} \tag{9}$$

set this partial derivative to 0 and solve for h. The solution for $h \in (0, 1)$ is[8]

$$h_{\min} = \frac{\sqrt{r_1 r_2}}{1 + \sqrt{r_1 r_2}}. \tag{10}$$

We plot this function in Figure 3 letting $r = r_1 = r_2$. When the witnesses are independent, claim (C) holds true for all the values of r and h underneath the curve.

[8] To assure ourselves that this solution yields a minimum rather than a maximum over r_1, $r_2 \in (0, 1)$, observe that the second derivative of h^* with respect to h is

$$\frac{\partial^2 h^*}{\partial h^2} = \frac{2 r_1 \bar{r}_1 r_2 \bar{r}_2}{(h + \bar{h} r_1 r_2)^3}$$

which is positive for r_1, $r_2 \in (0, 1)$.

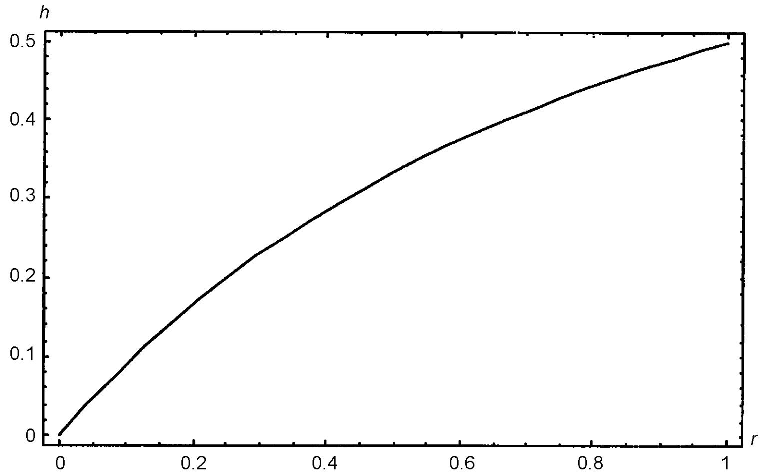

Figure 3. The space under the curve represents all points in which the posterior probability of the hypothesis is a decreasing function of the prior probability of the hypothesis when the witnesses are independent.

Subsequently, we turn to decision-making under uncertainty. Again, we assume a uniform distribution over r_1, r_2 and calculate the expected value of h^* for each value of h:

$$\langle h^* \rangle = \int_0^1 \int_0^1 \frac{(h + \bar{h} r_1)(h + \bar{h} r_2)}{h + \bar{h} r_1 r_2} dr_1 \, dr_2 \tag{11}$$

The analytical expression of this function is quite complex. We have plotted $\langle h^* \rangle$ in Figure 4. We now take the derivative of $\langle h^* \rangle$ with respect to h, set this derivative equal to 0 and solve for $h \in (0, 1)$:

$$h_{\min} \approx 0.199 \tag{12}$$

Notice that the area under the curve in Figure 3 is smaller than that in Figure 1: there is a smaller range of values of r and h for which claim (C) holds, when the witnesses are genuinely independent, than when they are either both reliable or both unreliable. Furthermore, when we have no clue how reliable the witnesses are, then claim (C) only holds true when there are more than 5 equiprobable suspects, i.e. when $h \leqslant 1/6$ and thus is less than 0.199. It is interesting to note that the conditions under which claim (C) holds up are actually more restrictive when we respect the stipulation that the witnesses are independent, rather than adopt Olsson's characterization of the witnesses.[9]

[9] If one wishes to compare Olsson's characterization of the witnesses with independent witnesses, it seems natural to keep the prior probability that a single witness is reliable fixed. However, if one would wish to keep the prior probability that *both* witnesses are reliable fixed to draw a comparison, see note 12.

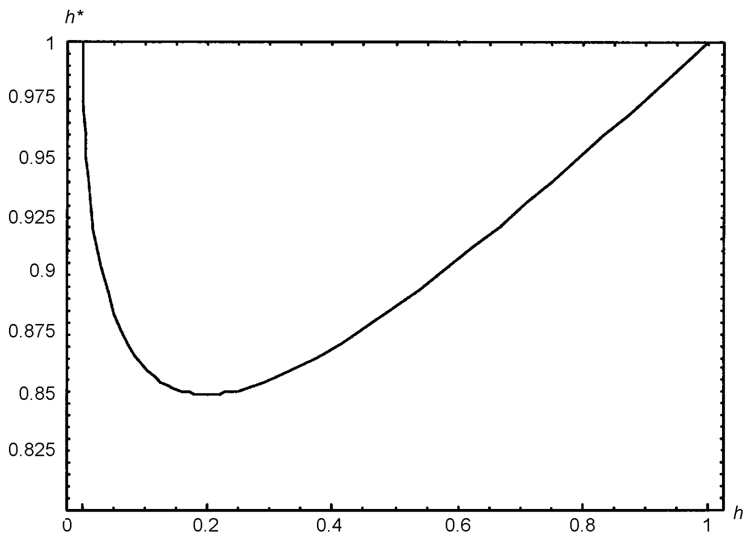

Figure 4. The posterior probability of the hypothesis after corroborating reports from two independent witnesses as a function of the prior probability of the hypothesis under conditions of decision-making under uncertainty.

The reader might wonder what happens when more than two independent witnesses all incriminate Forbes. For independent witnesses, the posterior probability that the hypothesis is true equals:

Theorem 4

$$P(\mathrm{H}|\mathrm{E}_1, \ldots, \mathrm{E}_n) = h^* = \frac{h}{h + \bar{h}\prod_{i=1}^{n} x_i}$$

with the likelihood ratios[10]

$$x_i = \frac{P(\mathrm{E}_i|\neg\mathrm{H})}{P(\mathrm{E}_i|\mathrm{H})} = \frac{h\bar{r}_i}{h + \bar{h}r_i}$$

for independent witnesses $i = 1, \ldots, n$.

Clearly, the posterior probability that Forbes committed the crime will rise as the number of witnesses rises. But what are the ranges of prior probabilities of the hypothesis for which Cohen's claim (C) holds true for values of $n = 3, 4, \ldots$? The posterior probability that the hypothesis is true when there are n independent witnesses is calculated in Theorem 4. We have

[10] One needs to be careful when talking about the likelihood ratio in Bayesian confirmation theory. Sometimes the likelihood ratio is defined as above (e.g. in Howson and Urbach [1993], p. 29), sometimes as the reciprocal of this ratio (e.g. in Pearl [1988], p. 34). We follow Howson and Urbach's terminology.

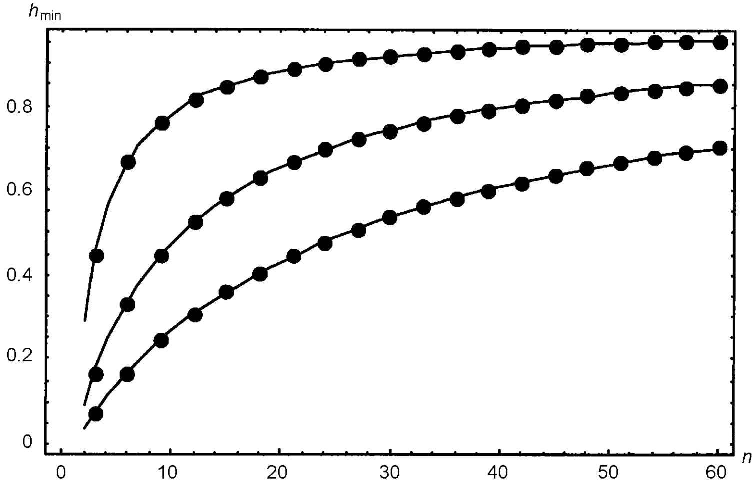

Figure 5. Claim (C) holds true for the ranges of prior probabilities of the hypothesis $(0, h_{\min})$ under decision-making under risk when there are n independent witnesses with $r_i = 0.04$ (bottom curve), $r_i = 0.1$ (middle curve) and r_i is 0.4 (top curve) for $i = 1, \ldots, n$.

calculated these values for decision-making under risk and for decision-making under uncertainty. For decision-making under risk,[11]

Theorem 5 If $r_i = r$ for all $i = 1, \ldots, n$, then claim (C) holds if and only if $h \in (0, h_{\min})$ with

$$h_{\min} = \frac{(n-1)r}{1+(n-1)r}.$$

We have plotted $h_{\min}$ as a function of n in Figure 5 for three different values of r. Notice that when r is 0.10, then claim (C) is true for *all* values of $h \in (0, 0.50)$ if and only if there are 11 or more corroborating reports. For witnesses who are less likely to be reliable, it takes more corroborating reports for claim (C) to hold over a wide range of values of h. For decision-making under uncertainty, we have plotted the function in Figure 6. We have shown in Theorem 6 in the Appendix how this plot can be obtained numerically. Notice that claim (C) is true for *all* values of $h \in (0, 0.50)$ when there are five or more corroborating reports. As the number of witnesses n providing corroborating reports increases, the range of values for which claim (C) holds broadens, and converges to (0, 1) as n converges to infinity.

[11] Clearly, for $n = 2$ and $r_i = r$ for $i = 1, 2$, $h_{\min}$ in Theorem 5 equals $h_{\min}$ in (10). The reader may also have noticed that if we make the assumption that the chance r that both witnesses are reliable in Olsson's model equals the chance $r_1 r_2$ that two independent witnesses in our model are reliable, then $h_{\min}$ in (5) equals $h_{\min}$ in (10). We would like to point out that this equality no longer holds if we generalize Olsson's model for n witnesses: there is no simple analytical formula that expresses $h_{\min}$ as a function of n and r.

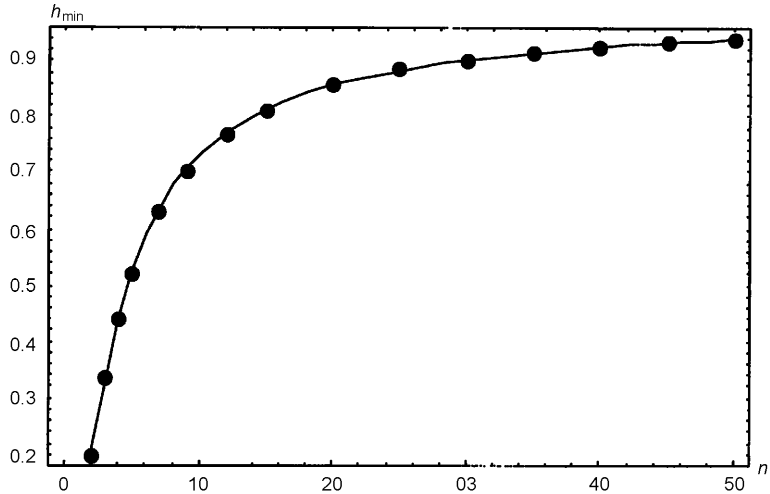

Figure 6. Claim (C) holds true for the ranges of prior probabilities of the hypothesis (0, h_{min}) when there are n independent witnesses under conditions of decision-making under uncertainty.

3 Robustness and multiple measures of corroboration

We now turn to claim (C*). Olsson takes the difference measure, i.e. $d(\mathrm{H}, \mathrm{E}) = P(\mathrm{H}|\mathrm{E}) - P(\mathrm{H})$, as his measure of corroboration. He shows that claim (C*) is false if we assume that we know how unreliable the witnesses are. To know how unreliable the witnesses are is to know $P(\mathrm{E}_i|\mathrm{H})$ and $P(\mathrm{E}_i|\neg\mathrm{H})$ for $i = 1, 2$, i.e. the probability that each witness will give us an incriminating report about Forbes conditional on Forbes having committed the crime and conditional on Forbes not having committed the crime, respectively. Subsequently he goes on to show that (C*) is true if we assume that the reliability of the witnesses is unknown: that we have a certain degree of confidence $r \in (0, 1)$ that the witnesses are both reliable and a certain degree of confidence that the witnesses are both unreliable. (We will continue to work in this section with Olsson's characterization of the relationship between the witnesses.) To establish his conclusion, he plots the difference measure as a function of the prior probability of the hypothesis. It turns out that this measure is indeed not a monotonically decreasing function (Olsson's Figure 4) if we assume that the degree of reliability of the witnesses is known, and is a monotonically decreasing function (Olsson's Figure 5) if we assume that the reliability of the witnesses is unknown.

This is an interesting result, but it is too soon to draw any conclusions. The problem is that there has been a long debate about what constitutes a proper measure of corroboration. (For surveys, see Eells and Fitelson [2002];

Fitelson [1999] and [2001a].) The difference measure certainly has received some support but it is by no means a privileged measure in the set of proposed measures. Furthermore these measures are extremely fickle in their behavior as a function of the prior probability of the hypothesis. To show how fragile Olsson's findings are, we will calculate the log-ratio measure, which has been defended by Milne ([1995]) for the case in which the reliability is known and the Carnap measure ([1962], Section 67) for the case in which the reliability is unknown.

For Olsson's argument to stand, the measure must be a function of the prior probability of the hypothesis that is *not* monotonically decreasing when the reliability of the witnesses is known and *is* a monotonically decreasing function when the reliability of the witnesses is unknown. The difference measure certainly provides him with this result. But a choice of other measures does not provide this result. As a matter of fact, we can turn Olsson's result around by varying the choice of confirmation measures. We will show that the log-ratio measure *is* a monotonically decreasing function when the reliability of the witnesses is known and that the Carnap measure *is* *not* a monotonically decreasing function when the reliability of the witnesses is unknown.

The log-ratio measure is defined as:

$$r(\mathrm{H},\,\mathrm{E}) = \ln\left(\frac{P(\mathrm{H}|\mathrm{E})}{P(\mathrm{H})}\right) \tag{13}$$

The Carnap measure is defined as:

$$c(\mathrm{H},\,\mathrm{E}) = P(\mathrm{E}\ \&\ \mathrm{H}) - P(\mathrm{E})P(\mathrm{H}) = P(\mathrm{E})(P(\mathrm{H}|\mathrm{E}) - P(\mathrm{H})) = P(\mathrm{E})d(\mathrm{H},\,\mathrm{E}) \tag{14}$$

Let us first consider the case of known reliability. In Olsson, the evidence E consists of two witness reports E_1 and E_2. We remind the reader of the relevant calculations in Olsson's paper. First, we present a more standard expression of Olsson's calculation of the posterior probability of the hypothesis when the reliability of the witnesses is known and make the following substitution for the likelihood ratio $x = P(\mathrm{E}_i|\neg\mathrm{H})/P(\mathrm{E}_i|\mathrm{H})$ for $i = 1, 2$:

$$P(\mathrm{H}|\mathrm{E}_1,\,\mathrm{E}_2) = \frac{P(\mathrm{H})}{P(\mathrm{H}) + \dfrac{P(\mathrm{E}_1|\neg\mathrm{H})}{P(\mathrm{E}_1|\mathrm{H})}\dfrac{P(\mathrm{E}_2|\neg\mathrm{H})}{P(\mathrm{E}_2|\mathrm{H})}P(\neg\mathrm{H})} = \frac{h}{h + \bar{h}x^2} \tag{15}$$

We can now insert the expression in (15) for known reliability into (13), i.e. into the log-ratio measure. Some algebraic manipulations yield:

$$r(\mathrm{H},\,\mathrm{E}) = -\ln(h + \bar{h}x^2) \tag{16}$$

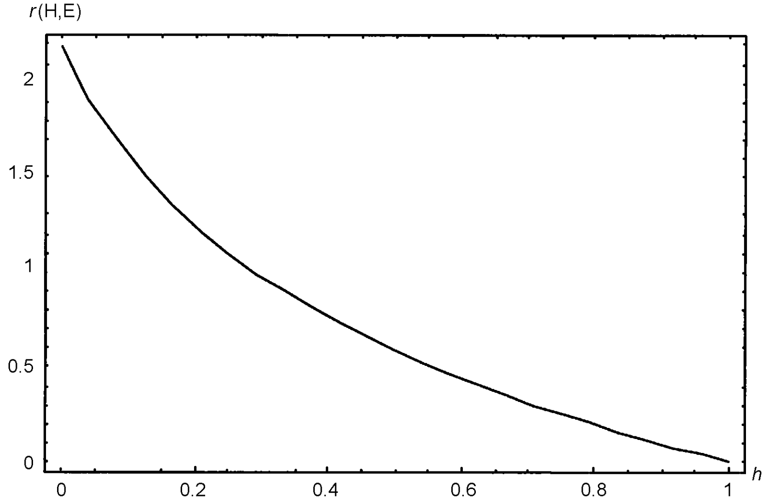

Figure 7. The log-ratio measure as a function of the prior probability of the hypothesis when the reliability of the witnesses is known for $x = 1/3$.

We have plotted this measure for $x = 1/3$ in Figure 7. Clearly, r(H, E) is a strictly monotonically decreasing function of h. More generally, since $x < 1$,[12]

$$\frac{\partial r(\mathrm{H},\ \mathrm{E})}{\partial h} = -\frac{1 - x^2}{h + \bar{h}x^2} < 0. \tag{17}$$

r(H, E) is a strictly monotonically decreasing function of h. Hence, the log-ratio measure does not support Olsson's argument to the effect that claim (C*) does not hold for known reliability. Furthermore, note that the log-ratio measure is not just an isolated measure: it is one of a class of ordinally equivalent measures which all yield the same result in this argument. Two other measures in the class are the ratio measure $P(\mathrm{H}|\mathrm{E})/P(\mathrm{H})$ and the normalized difference measure $(P(\mathrm{H}|\mathrm{E}) - P(\mathrm{H}))/P(\mathrm{H}) = d(\mathrm{H},\ \mathrm{E})/P(\mathrm{H})$.

Second, we consider Olsson's case of unknown reliability. In this case, $P(\mathrm{H}|\mathrm{E})$ is h^* in (3). Olsson presents P(E) in his formula (U5). In our notation:

$$P(\mathrm{E}) = hr + h^2\bar{r} \tag{18}$$

We insert the expressions in (3) and (18) into (14), i.e. into the Carnap measure, and obtain

$$c(\mathrm{H},\ \mathrm{E}) = h\bar{h}r. \tag{19}$$

We have plotted this measure for $r = 0.10$ in Figure 8. Since

[12] Note that E is evidence for H and hence $P(\mathrm{H}|\mathrm{E}) > P(\mathrm{H})$. This is equivalent to $x < 1$.

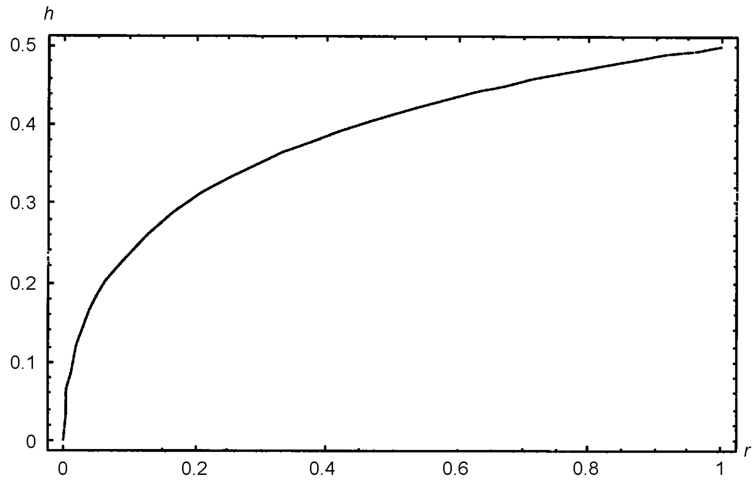

Figure 8. The Carnap measure as a function of the prior probability of the hypothesis when the reliability of the witnesses is unknown for $r = 0.10$.

$$\frac{\partial c(\mathrm{H, R})}{\partial h} = (1 - 2h)r. \tag{20}$$

$c(\mathrm{H, E})$ is monotonically increasing for $h < 0.5$ and monotonically decreasing for $h > 0.5$. Hence, the Carnap measure does not support Olsson's argument to the effect that claim (C*) holds for unknown reliability.[13]

Olsson's argument to the effect that claim (C*) does not hold for known reliability and does hold for unknown reliability is contingent on the endorsement of the difference measure as a proper measure of corroboration. Since the argument does not hold when we invoke certain alternative measures, he needs to present an independent argument that the difference measure is indeed a preferable measure of corroboration.

Alternatively, there is also another tack that is open to Olsson. The log-odds ratio has been fiercely defended by I. J. Good ([1983]) and a new pitch for the measure has been made by B. Fitelson ([1999] and [2001a]):

$$o(\mathrm{H, E}) = \ln\left(\frac{O(\mathrm{H|E})}{O(\mathrm{H})}\right) \tag{21}$$

with the posterior odds $O(\mathrm{H|E}) = P(\mathrm{H|E})/(1 - P(\mathrm{H|E}))$ and the prior odds $O(\mathrm{H}) = P(\mathrm{H})/(1 - P(\mathrm{H}))$. We do not want to take a position in the debate about which measure of corroboration is superior, but it is worth pointing out that Olsson's argument also stands if we endorse the log-odds ratio. First,

[13] Olsson's argument also breaks down on the Nozick measure ([1981], p. 252) $n(\mathrm{H, E}) = P(\mathrm{E|H}) - P(\mathrm{E|\neg H})$. Following our methodology, it can be shown that $n(\mathrm{H, E})$ is a constant function of h for unknown reliability in Olsson's model.

calculate the log-odds ratio for known reliability by inserting $P(\mathrm{H}|\mathrm{E}) = P(\mathrm{H}|\mathrm{E1, E2})$ as calculated in (15) into (21):

$$o(\mathrm{H, E}) = \ln(1/x^2) \tag{22}$$

Clearly, $o(\mathrm{H, E})$ is independent of H and hence a constant function of the prior probability h. This is sufficient for Olsson: for known reliability, $o(\mathrm{H, E})$ is *not* a decreasing function of h.

Second, for unknown reliability, we calculate the log-odds ratio by inserting $P(\mathrm{H}|\mathrm{E}) = h^*$ as calculated in (3) into (22). Some algebraic manipulations yield:

$$o(\mathrm{H, E}) = \ln\left(\frac{r + h^2\bar{r}}{h^2\bar{r}}\right) \tag{23}$$

We calculate the partial derivative with respect to h:

$$\frac{\partial o(\mathrm{H, E})}{\partial h} = -\frac{2r}{h(r + h^2\bar{r})} \tag{24}$$

which is clearly smaller than 0. Hence, when the reliability of the witnesses is unknown, the log-odds ratio *is* a decreasing function of the prior probability of h, which is sufficient for Olsson.

To sum up, Olsson's argument is contingent on either an endorsement of the difference measure or the log-odds ratio (or some other relevance measure with the desired properties). It does not hold on the log-ratio measure, the ratio measure, the normalized ratio measure, the Carnap measure, or the Nozick measure.

4 Discussion

There is an interesting history to the problem that Olsson puts his finger on and to the model that he develops to address this problem. The idea of treating the reliability of witnesses as an endogenous variable is already present in C. I. Lewis. Lewis ([1946], p. 346) remarks that if we receive the same item of information from multiple independent witnesses, then we become more confident that the witnesses are truth-tellers. The idea of modeling the reliability of the 'evidentiary mechanism', say a witness, as an endogenous variable in a model of hypothesis confirmation is also the central idea of what D. A. Schum ([1988], pp. 258–61) names the 'Scandinavian School of Evidentiary Value'. Within this school, Hansson ([1983], p. 78), with reference to Edman ([1973]), imposes the following condition on unreliable witnesses: conditional on a witness being unreliable, the hypothesis and the evidence are independent. This statement is tantamount to the part of conditions (iii) and (iv) in Olsson which states that (*)

$P(\mathrm{E}|\mathrm{H}, \mathrm{U}) = P(\mathrm{E}|\neg\mathrm{H}, \mathrm{U})$. Bonjour ([1985]) takes inspiration from the C. I. Lewis passage in order to support his version of a coherence theory of justification, and it is through Bonjour that (*) enters into Huemer ([1997]) and into Bovens and Olsson ([2000]). Also within the Scandinavian school, it is Ekelöf ([1983], p. 22) who at least hints in the direction of claim (C). He proposes a particular formula to calculate our degree of confidence after concurring reports, but concedes that when two witnesses report something 'quite extraordinary', then each report may count very little in the way of evidence, but both reports together raise our degree of confidence that what is reported is true beyond the value which the formula yields.

The idea in (*) is that if witnesses are fully unreliable, then they do not even look at the world to provide a report that H is the case. They act no differently from randomizers. It is as if they flip a coin or roll a die to decide whether to report that H is the case, and hence whether H is or is not the case is of no import. The chance that they report that H is the case, i.e. the level at which they are randomizing, ranges over the open interval (0, 1). In Bovens and Hartmann's model ([2002], p. 32) of hypothesis confirmation with unreliable instruments, this chance is the value of a randomization parameter a. Olsson's crucial insight is the following. Fully unreliable witnesses are witnesses that did not see anything and hence can do no better than we could. We know that Forbes is one out of n equiprobable suspects, and hence the chance that he did it is $h = 1/n$, i.e. the prior probability of the hypothesis. So the chance that an unreliable suspect will pick Forbes is exactly $P(\mathrm{E}|\mathrm{H}, \neg\mathrm{R}) = P(\mathrm{E}|\neg\mathrm{H}, \neg\mathrm{R}) = h$. This is what leads to Olsson's result that the posterior probability of H is a decreasing function of the prior probability of H for certain ranges of prior probabilities that the witnesses are fully unreliable.

What is important to realize is that Olsson's result does not occur when we merely assume that the reliability of the witnesses is unknown without setting the randomization parameter at h. The reliability of the witnesses is just as unknown when we stipulate that the witnesses may or may not be fully reliable, and if they are fully unreliable, then they flip a coin to determine whether they will report that H is true, i.e. $P(\mathrm{E}|\mathrm{H}, \neg\mathrm{R}) = P(\mathrm{E}|\neg\mathrm{H}, \neg\mathrm{R}) = 1/2$. If we impose this stipulation on Olsson's model, then the posterior probability will simply be an increasing function of the prior probability for all values of the variable r.

So, how reasonable is it to set the randomization parameter at the prior probability of the hypothesis h? This stipulation is contingent on the following two assumptions, viz. (i) all the alternative hypotheses are equally probable and (ii) an unreliable witness acts as if he knows no more than we do. This story can be made plausible for a lineup of suspects. It may well be the case that we take it to be equally probable for each suspect to be the criminal. And it may well be the case that fully unreliable witnesses act as if they are randomizers, with the chance that an arbitrary suspect is picked being $1/n$. But presumably, concurrence in witness reports concerning a 'quite extraordinary' option also constitutes strong evidence, even if there is no uniform distribution over the alternatives. For instance, suppose that you are shopping around for some specialized consumer product, say a bread machine, a DVD player, or what have you. There is *one* better-known and a range of less well-known brand names on the market. It is shared knowledge that better products tend to come from better-known brand names, but there are many exceptions to this rule. Hence, the prior probability that the better product comes from one of the better-known brand names exceeds the prior probability that it comes from one of the less well-known brand names. You are soliciting advice from independent sources. It is not clear to you who really knows anything more than you do about the consumer product in question. Now compare two cases. In case one, all your sources recommend the better-known brand name. In case two, all your sources recommend a very obscure brand name. In what case is your degree of confidence greater that the recommended product is the better product? Presumably 'too odd not to be true' reasoning should carry some weight here as well. Sometimes, I *will* find myself more convinced by the concurring recommendations of the less well-known product, since the recommendations of the better-known product give me no reason to believe that my informers know any more than I do.

A general model of 'too odd not to be true' reasoning should also be capable of handling cases in which there is no uniform distribution over the possible alternatives. But is it plausible in this case to stipulate that $P(\mathrm{E}|H, \neg\mathrm{R}) = P(\mathrm{E}|\neg\mathrm{H}, \neg\mathrm{R}) = h$? There seems to be no justification to build this assumption into the model. We could stipulate the idealization that an unreliable witness is fully ignorant and will just randomize over the n alternative products with probability $1/n$. An alternative idealization is that an unreliable witness knows precisely what we know and will just pick the more probable alternative, i.e. the product from the better-known brand name. But we do not see how one could argue in this case that an unreliable witness randomizes over the alternatives so that the chance of picking some alternative matches our prior probability distribution over the alternatives.

We conclude that Olsson's insight to set the randomization parameter for unreliable witness equal to the prior probability of the hypothesis provides an interesting route towards giving an account of 'too odd not to be true' reasoning in a restricted range of cases. However, as soon as we give up the assumptions of a uniform prior distribution over the alternatives, it remains an open question as to how to model this type of reasoning in a Bayesian framework.

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Appendix: Proofs of Theorems

A. Proof of Theorem 1

Theorem 1 If (i) through (viii), then $P(\mathrm{E}_i|\mathrm{H}, \mathrm{E}_j) > P(\mathrm{E}_i|\mathrm{H})$ for $i, j = 1, 2$ and $i \neq j$. We prove that $P(\mathrm{E}_2|\mathrm{H}, \mathrm{E}_1) > P(\mathrm{E}_2|\mathrm{H})P(\mathrm{E}_1|\mathrm{H}, \mathrm{E}_2) > P(\mathrm{E}_1|\mathrm{H})$ is analogous.

Proof:

(1) $P(\mathrm{E}_2|\mathrm{H}) = P(\mathrm{E}_2|\mathrm{H}, \mathrm{U})P(\mathrm{U}|\mathrm{H}) + P(\mathrm{E}_2|\mathrm{H}, \mathrm{R})P(\mathrm{R}|\mathrm{H})$

$= P(\mathrm{H})P(\mathrm{U}) + P(\mathrm{R})$ by (i), (iii), and (vii), (viii)

(2) $P(\mathrm{E}_1, \mathrm{E}_2|\mathrm{H}) = \dfrac{P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2)P(\mathrm{E}_1, \mathrm{E}_2)}{P(\mathrm{H})}$ by the Prob. Calc. (PC)

$= P^2(\mathrm{H})P(\mathrm{U}) + P(\mathrm{R})$ from Olsson's (U5) and (U6)

(3) $P(\mathrm{E}_2|\mathrm{E}_1, \mathrm{H}) = \dfrac{P(\mathrm{E}_1, \mathrm{E}_2|\mathrm{H})}{P(\mathrm{E}_1|\mathrm{H})}$ by PC

$= \dfrac{P^2(\mathrm{H})P(\mathrm{U}) + P(\mathrm{R})}{P(\mathrm{H})P(\mathrm{U}) + P(\mathrm{R})}$ from (1) and (2)

(4) $P(\mathrm{E}_2|\mathrm{E}_1, \mathrm{H}) - P(\mathrm{E}_2|\mathrm{H}) = \dfrac{(1 - P(\mathrm{H}))^2 P(\mathrm{R})P(\mathrm{U})}{P(\mathrm{R}) + P(\mathrm{H})P(\mathrm{U})} > 0$ from (1) and (3)

B. Proof of Theorem 2

Theorem 2 If (i) through (viii), then $P(\mathrm{E}_i|\neg\mathrm{H}, \mathrm{E}_j) > P(\mathrm{E}_i|\neg\mathrm{H})$ for $i, j = 1, 2$ and $i \neq j$. We prove that $P(\mathrm{E}_2|\neg\mathrm{H}, \mathrm{E}_1) > P(\mathrm{E}_2|\neg\mathrm{H})$. $P(\mathrm{E}_1|\neg\mathrm{H}, \mathrm{E}_2) > P(\mathrm{E}_1|\neg\mathrm{H})$ is analogous.

Proof:

(1) $P(\mathrm{E}_2|\neg\mathrm{H}) = P(\mathrm{E}_2|\neg\mathrm{H}, \mathrm{U})P(\mathrm{U}|\neg\mathrm{H}) + P(\mathrm{E}_2|\neg\mathrm{H}, \mathrm{R})P(\mathrm{R}|\neg\mathrm{H})$

$= P(\mathrm{H})P(\mathrm{U})$ by (ii), (iv), and (viii)

(2) $P(\mathrm{E}_1, \mathrm{E}_2|\neg\mathrm{H}) = \dfrac{(1 - P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2))P(\mathrm{E}_1, \mathrm{E}_2)}{1 - P(\mathrm{H})}$ by PC

$= P^2(\mathrm{H})P(\mathrm{U})$ from Olsson's (U5) and (U6)

(3) $P(\mathrm{E}_2|\mathrm{E}_1, \neg\mathrm{H}) = \dfrac{P(\mathrm{E}_1, \mathrm{E}_2|\neg\mathrm{H})}{P(\mathrm{E}_1|\neg\mathrm{H})}$ by PC

$= P(\mathrm{H})$ from (1) and (2)

(4) $P(\mathrm{E}_2|\mathrm{E}_1, \neg\mathrm{H}) - P(\mathrm{E}_2|\neg\mathrm{H}) = P(\mathrm{H}) - P(\mathrm{H})P(\mathrm{U}) = P(\mathrm{H})P(\mathrm{R}) > 0$

from (1) and (3)

C. Proof of Theorem 3

Theorem 3

$$P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = h^* = \frac{(h+\bar{h}r_1)(h+\bar{h}r_2)}{h+\bar{h}r_1 r_2}$$

Proof:

We calculate the posterior probability that H is true after having learned two independent incriminating witness reports. Let $P(\mathrm{R}_i) = r_i$ be the chance that witness i is reliable and $P(\mathrm{U}_i) = 1 - r_i$. We adopt the independence conditions in (1) and (2) and transpose the following assumptions from Olsson within the context of independent voters: for $i = 1, 2$, (i′) $P(\mathrm{E}_i|H, \mathrm{R}_i) = 1$; (ii′) $P(\mathrm{E}_i|\neg\mathrm{H}, \mathrm{R}_i) = 0$; (iii′) $P(\mathrm{E}_i|\mathrm{H}, \mathrm{U}_i) = P(\mathrm{H})$; (iv′) $P(\mathrm{E}_i|\neg\mathrm{H}, \mathrm{U}_i) = P(\mathrm{H})$; (vii′) $P(\mathrm{R}_i|\mathrm{H}) = P(\mathrm{R}_i) = P(\mathrm{R}_i|\neg\mathrm{H})$; (viii′) $P(\mathrm{U}_i|\mathrm{H}) = P(\mathrm{U}_i) = P(\mathrm{U}_i|\neg\mathrm{H})$. The probability distribution over the random variables H, R_1, R_2 is non-extreme.

(1) $$P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = \frac{P(\mathrm{H})}{P(\mathrm{H}) + \frac{P(\mathrm{E}_1, \mathrm{E}_2|\neg\mathrm{H})}{P(\mathrm{E}_1, \mathrm{E}_2|\mathrm{H})}P(\neg\mathrm{H})} \quad \text{by PC}$$

(2) $$= \frac{P(\mathrm{H})}{P(\mathrm{H}) + \frac{P(\mathrm{E}_1|\neg\mathrm{H})}{P(\mathrm{E}_1|\mathrm{H})}\frac{P(\mathrm{E}_2|\neg\mathrm{H})}{P(\mathrm{E}_2|\mathrm{H})}P(\neg\mathrm{H})} \quad \text{by independence}$$

Note that when the witnesses are independent, the likelihood ratio can be factorized, while this factorization does not hold in Olsson's analysis.

(3) $$P(\mathrm{E}_i|\mathrm{H}) = P(\mathrm{E}_i|\mathrm{R}_i, \mathrm{H})P(\mathrm{R}_i|\mathrm{H}) + P(\mathrm{E}_i|\mathrm{U}_i, \mathrm{H})P(\mathrm{U}_i|\mathrm{H}) \quad \text{by PC}$$
$$= r_i + h\bar{r}_i \quad \text{by (i′), (iii′) (vii′) and (viii′)}$$

(4) $$P(\mathrm{E}_i|\neg\mathrm{H}) = P(\mathrm{E}_i|R_i, \neg\mathrm{H})P(\mathrm{R}_i|\neg\mathrm{H}) + P(\mathrm{E}_i|\mathrm{U}_i, \neg\mathrm{H})P(\mathrm{U}_i|\neg\mathrm{H}) \quad \text{by PC}$$
$$= h\bar{r}_i \quad \text{by (ii′), (iv′) (vii′) and (viii′)}$$

(5) $$P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = \frac{h}{h + \bar{h}\frac{h\bar{r}_1}{h+\bar{h}r_1}\frac{h\bar{r}_2}{h+\bar{h}r_2}} \quad \text{from (2), (3) and (4)}$$

This can be simplified to:

(6) $$P(\mathrm{H}|\mathrm{E}_1, \mathrm{E}_2) = \frac{(h+\bar{h}r_1)(h+\bar{h}r_2)}{h+\bar{h}r_1 r_2}$$

D. Proof of Theorem 4

Theorem 4

$$P(\mathrm{H}|\mathrm{E}_1, \ldots, \mathrm{E}_n) = h^* = \frac{h}{h + \bar{h}\prod_{i=1}^{n} x_i}$$

with the likelihood ratios

$$x_i = \frac{h\bar{r}_i}{h + \bar{h}r_i}$$

for independent witnesses $i = 1, \ldots, n$.

Proof:

The proof follows by a straightforward generalization of Theorem 3.

E. Proof of Theorem 5

Theorem 5 If $r_i = r$ for all $i = 1, \ldots, n$, then claim (C) holds if and only if $h \in (0, h_{\min})$ with

$$h_{\min} = \frac{(n-1)r}{1 + (n-1)r}.$$

Proof:

If all $r_i = r$, Theorem 4 simplifies to:

(1) $P(\mathrm{H}|\mathrm{E}_1, \ldots, \mathrm{E}_n) = h^* = \dfrac{h}{h + \bar{h}x^n}$ with $x = \dfrac{h\bar{r}}{h + \bar{h}r}$

Differentiating h^* with respect to h, one obtains:

(2) $\dfrac{\partial h^*}{\partial h} = \left(\dfrac{h^*}{h}\right)^2 x^n \left(1 - \dfrac{nr\bar{h}}{r + h\bar{r}}\right)$

We set this partial derivative to 0 and solve for $h \in (0, 1)$:

(3) $h_{\min} = \dfrac{(n-1)r}{(n-1)r + 1}$

By examining the second derivative of h^* with respect to h, it can be shown that this value of h corresponds to the minimum of h^*.

F. Proof of Theorem 6

In order to plot Figure 6, we used the following theorem:

Theorem 6 For $h > 0.50$, the expected value of h^* for n independent witnesses is

$$\langle h^* \rangle = \sum_{k=0}^{\infty} (-1)^k \left(\frac{\bar{h}}{h}\right)^k I_k^n(h) \text{ with } I_k(h) = \int_0^1 \frac{hx^k dx}{(h + \bar{h}x)^2}.$$

Proof:

Introducing new integration variables:

$$(1) \quad x_i = \frac{h\bar{r}_i}{h + \bar{h}r_i}$$

one obtains from Theorem 4:

$$(2) \quad \langle h^* \rangle = \int_0^1 dr_1 \ldots \int_0^1 dr_n h^* = \int_0^1 \ldots \int_0^1 \frac{1}{1 + \frac{\bar{h}}{h}\prod_{i=1}^{n} x_i} \prod_{j=1}^{n} \frac{h dx_j}{(h + \bar{h}x_j)^2}$$

Since $0 < x_i < 1$, the expression

$$\frac{\bar{h}}{h}\prod_{i=1}^{n} x_i < 1$$

iff $h > 0.50$. Hence, the first expression under the integral can be expanded in an infinite sum:

$$(3) \quad \frac{1}{1 + \frac{\bar{h}}{h}\prod_{i=1}^{n} x_i} = \sum_{k=0}^{\infty}(-1)^k \left(\frac{\bar{h}}{h}\right)^k \prod_{i=1}^{n} x_i^k \text{ for } h > 0.50$$

Substituting (3) in (2), we obtain:

$$(4) \quad \langle h^* \rangle = \sum_{k=0}^{\infty}(-1)^k \left(\frac{\bar{h}}{h}\right)^k \prod_{i=1}^{n} \int_0^1 \frac{h x_i^k dx_i}{(h + \bar{h}x_i)^2}$$

$$= \sum_{k=0}^{\infty}(-1)^k \left(\frac{\bar{h}}{h}\right)^k \left[\int_0^1 \frac{h x^k dx}{(h + \bar{h}x)^2}\right]^n$$

To determine the minimum of $\langle h^* \rangle$, we did the exact n-dimensional numerical integration for $n = 2$, 3, 4 and 5, using *Mathematica* and applied the built-in function **FindMinimum**. Following this procedure, the computation time grows exponentially with n. For $n > 5$, the minimum turns out to be at values of $h > 0.50$, and the above expression for $\langle h^* \rangle$ can be used. The integrals $I_k(h)$ can easily be computed recursively. Note that the computation time is now independent of n.