GRW Theory

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Consider a toy system consisting of a marble and box. The marble has two sates, $|\Psi_{in}\rangle$ and $|\Psi_{out}\rangle$, corresponding to the marble being inside or outside the box. These states are eigenvectors of the operator $\hat{B}$, measuring whether the marble is inside or outside the box. The formalism of quantum mechanics (QM) has it that not only $|\Psi_{in}\rangle$ and $|\Psi_{out}\rangle$ themselves, but any superposition $|\Psi_m\rangle = a|\Psi_{in}\rangle + b|\Psi_{out}\rangle$ where a and b are complex numbers such that $|a|^2 + |b|^2 = 1$, can be the state of the marble. What are the properties of the marble in such a state? This question is commonly answered by appeal to the so-called Eigenstate-Eigenvalue Rule (EER): An observable O has a well-defined value for a quantum a system S in state $|\Psi\rangle$ if, and only if, $|\Psi\rangle$ is an eigenstate of O. Since $|\Psi_{in}\rangle$ and $|\Psi_{out}\rangle$ are eigenstates of $\hat{B}$, EER yields that the marble is either inside (or outside) the box if its state is $|\Psi_{in}\rangle$ (or $|\Psi_{out}\rangle$). However, states like $|\Psi_m\rangle$ defy interpretation on the basis of EER and we have to conclude that if the marble is in such a state then it is neither inside nor outside the box. This is unacceptable because we know from experience that marbles are always either inside or outside boxes. Reconciling this fact of everyday experience with the quantum formalism is the infamous measurement problem. Standard quantum mechanics solves this problem, following a suggestion of von Neumann's, by postulating that upon measurement the system's state is instantaneously reduced to one of the eigenstates of the measured observable, which leaves the system in a state that can be interpreted on the basis of EER (see Measurement Theory). However, it is generally accepted that this proposal is ultimately unacceptable. What defines a measurement? At what stage of the measurement process does the collapse take place (trigger problem)? And why should the properties of a system depend on actions of observers?

GRW Theory (sometimes also ‘GRW model’) is a suggestion to overcome these difficulties (Ghirardi, Rimini, and Weber (1986); Bell (1987) and Ghirardi (2002) provide short and non-technical presentations of the theory; for a comprehensive discussion of the entire research programme to which GRW Theory belongs see Bassi and Ghirardi (2003)). The leading idea of the theory is to eradicate observers from the picture and view state reduction as a process that occurs as a consequence of the basic laws of nature. The theory achieves this by adding to the fundamental equation of QM, the Schrödinger equation, a stochastic term which describes the state reduction occurring in the system. (For this reason GRW theory is not, strictly speaking, an interpretation of QM; it is a quantum theory in its own right).

A system governed by GRW theory evolves according to the Schrödinger equation all the time except when a state reduction, a so-called hit, occurs (hits are also referred to as ‘hittings’, ‘perturbations’, ‘spontaneous localisations’, ‘collapses’, and ‘jumps’). A crucial assumption of the theory is that hits occur at the level of the micro constituents of a system (in the above example at the level of the atoms that make up the marble). The crucial question then is: when do hits occur and what exactly happens when they occur?

GRW Theory posits that the occurrence of hits constitutes a Poisson process. Generally speaking, Poisson processes are processes characterised in terms of the number of occurrences of a particular type of event in a certain interval of time τ, for instance the number of people passing through a certain street during time τ. These events are Poisson distributed if the probability that the number of events occurring during τ, n, takes value m is given by $p(n=m)=e^{-\lambda\tau}(\lambda\tau)^m/m!$, where λ is the parameter of the distribution. One can show that λ is also the mean value of the distribution and hence it can be interpreted as the average number of events occurring per unit time. GRW theory sets $\lambda=10^{-16}s^{-1}$ and posits that this is a new constant of nature. Hence, in a macroscopic system that is made up of about 10^{23} atoms there are on average 10^{7} hits per second.

A hit transforms the system’s state into another state according to a probabilistic algorithm that takes the position basis as the privileged basis (in that the reduction process leads to a localisation of the system’s state in the position basis). Let $|\Psi_S\rangle$ be state of the entire system (e.g. the marble) before the hit occurs. When the k^{th} particle, say, is hit the state is instantaneously transformed into another, more localised state:

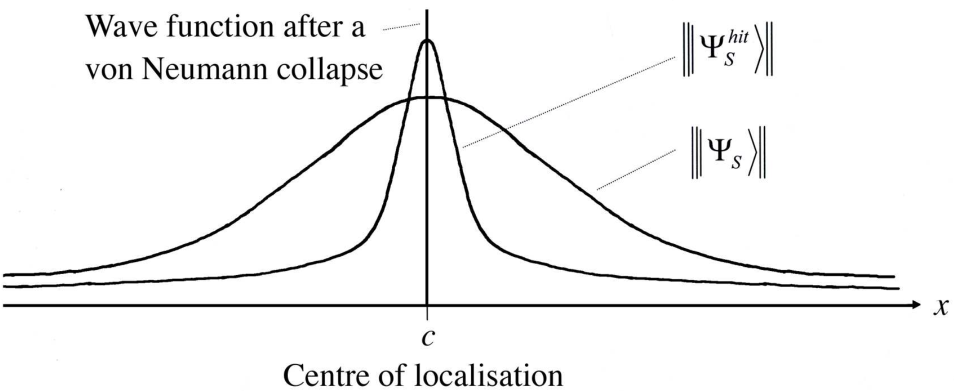
$$\left|\Psi_S\right\rangle \rightarrow \left|\Psi_S^{hit}\right\rangle = \frac{L_{k,c}\left|\Psi_S\right\rangle}{\left\|L_{k,c}\left|\Psi_S\right\rangle\right\|}.$$

$L_{k,c}$, the localisation operator, that has the shape of Gaussian (a bell-shaped curve) centred around c, which is chosen at random according to the distribution $p_k(c) = \left\|L_{k,c}\left|\Psi_S\right\rangle\right\|^2$; the width σ of the Gaussian is also a new constant of nature, and it is of the magnitude $10^{-7}m$. The choice of this distribution assures that the predictions of GRW Theory coincide almost always with those of standard QM (there are domains in which the two theories do not yield the same predictions, but these are (so far) beyond the reach of experimental test; see Rimini (1995)).

Due to the mathematical structure of QM (more specifically, due to the fact that $|\Psi_S\rangle$ is the tensor product of the states of all its micro constituents) the hits at the micro level 'amplify': if the marble is in state $\left|\Psi_m\right\rangle$ and k^{th} particle gets hit, then the entire state is transformed into a highly localised state, i.e. all terms except one in the superposition are suppressed. This is GRW's solution of the measurement problem. A macro system gets hit 10^7 times per second and hence superpositions are suppressed almost immediately; micro systems are not hit very often and hence retain their 'quantum properties' for a very long time.

This proposal faces two important formal problems. First, the wave function of systems of identical particles has to be either symmetrical (in the case of Bosons) or antisymmetrical (in the case of Fermions), and remain so over the course of time. GRW theory violates this requirement in that wave functions that are symmetric (or antisymmetric) at some time need not be (and generally are not) symmetric (or antisymmetric) at later times. Second, although hits occur at the level of the system's wave function, the fundamental equation of the theory is expressed in terms of the density matrix. This strikes physicists as odd and as one would like to have an equation governing the evolution of the wave function itself. Both difficulties are overcome within the so-called CSL model (for 'continuous spontaneous localization') introduced in Pearle (1989) and Ghirardi, Pearle Rimini (1990). The model belongs to the same family of proposals as GRW theory in that it proposes to solve the measurement problem by an appeal to a spontaneous localisation processes. The essential difference is that the discontinuous hits of GRW theory are replaced by a continuous stochastic evolution of the state vector in Hilbert space (similar to a diffusion process).

Another serious problem concerns the nature of GRW hits. Unlike the state reduction that von Neumann introduced into standard QM, the hits of GRW theory do not leave the system's state in an *exact* position eigenstate; the post-hit state is highly peaked, but nevertheless fails to be a precise position eigenstate. This is illustrated schematically in Figure 1. Hence, strictly speaking the post-hit states are not interpretable on the basis of EER and we are back where we started. Common wisdom avoids this conclusion by pointing out that GRW post-hit states are close to eigenstates and positing that being close to an eigenstate is as good as being an eigenstate. This has been challenged by Lewis (1997), who presents an argument, now commonly referred to as the 'counting anomaly', for the conclusion that because of the failure of GRW hits to leave the system in a precise position eigenstate, GRW theory implies that arithmetic does not apply to macroscopic objects. For a critical discussion of this argument see Frigg (2003).



What is the correct interpretation of the theory? That is, what, if anything, does the theory describe? The answer to this question is less obvious than it might seem. Clifton and Monton (1999) regard it as a 'wave function only theory' according to which the world literally is just the wave function that the theory describes. Monton (2004) later criticises this view as mistaken and suggests a variation on the mass densitiy interpretation originally proposed by Ghirardi, Grassi, and Benatti (1995) as the right interpretation. Lewis (2005) points out that all versions of the mass density interpretation lead to a violation of common sense should hence not be regarded as a problem-free alternative.

How should we interpret the probabilities that the theory postulates in its hit mechanism? Are they best interpreted as propensities, frequencies, Humean chances, or yet something else? Or should the quest for such an interpretation be rejected as ill-conceived? This question is discussed in Frigg and Hoefer (2007) who come to the conclusion that GRW probabilies can be understood either as single case propensities or as Humean chances, while all other options are ruled out by GRW Theory itself.

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