

# Welfarism and the assessment of social decision rules

Claus Beisbart and Stephan Hartmann

## Abstract

The choice of a social decision rule for a federal assembly affects the welfare distribution within the federation. But which decision rules can be recommended on welfarist grounds? In this paper, we focus on two welfarist desiderata, viz. (i) maximizing the expected utility of the whole federation and (ii) equalizing the expected utilities of people from different states in the federation. We consider the European Union as an example, set up a probabilistic model of decision making and explore how different decision rules fare with regard to the desiderata. We start with a default model, where the interests, and therefore the votes of the different states are not correlated. This default model is then abandoned in favor of models with correlations. We perform computer simulations and find that decision rules with a low acceptance threshold do generally better in terms of desideratum (i), whereas the rules presented in the Accession Treaty and in the (still unratified) Constitution of the European Union tend to do better in terms of desideratum (ii). The ranking obtained regarding desideratum (i) is fairly stable across different correlation patterns.

## 1 Introduction

For a long time, social choice theory has been dominated by axiomatic approaches in the tradition of Arrow ([1]) and Sen ([9]). These works typically start with a few axioms that put intuitively reasonable constraints on the social welfare function, for instance. Unfortunately, it turns out that these constraints cannot be fulfilled at the same time. Impossibility results of this kind are very exciting. But they are of no help, if we are to decide between different social decision rules.

Consider the European Union as an example. Many decisions are taken by the European Council of Ministers (Council, for short). It works in the following way: Each state of the European Union sends a representative to the Council. The European Commission drafts a proposal, and the representatives cast their votes on behalf of the states. The votes are aggregated, and a decision is taken according to some decision rule. But which rule is most appropriate? Impossibility results do not answer this question.

In this paper, we will take a different line of thought. We will start with simple principles that spell out what makes a decision rule pro tanto better than another one. We will then evaluate decision rules according to these principles. As we will see, this requires us to set up a different framework (see [4]); and

we will need to use new mathematical techniques and computational methods such as computer simulations.

We choose a *welfarist* framework to evaluate alternative decision rules. It is based on the following simple idea. The outcomes of a decision affect the welfares of the people in the federation. A particular outcome may benefit some people, it may harm other people, and it may make no difference to yet others. Now different decision rules lead to different outcomes. As a consequence, different decision rules result in different welfare distributions.

But which decision rule is best? To address this question, the welfare distribution that results from the adoption of a certain decision rule has to be evaluated, and we propose to evaluate it according to the following two welfarist principles:

**Utilitarianism** Decision rule  $D_1$  is pro tanto better than decision rule  $D_2$ , if the expected utility is larger under  $D_1$  than under  $D_2$  (cp. [4]).

**Egalitarianism** Decision rule  $D_1$  is pro tanto better than decision rule  $D_2$ , if there is more equality in the distribution of the expected utilities across the federation under  $D_1$  than under  $D_2$ .<sup>1</sup>

We consider the European Union as an example. Over the last years, there has been a lot of controversy about the question which decision rule to adopt for the Council of Ministers (see, for example, [7]). Various decision rules have been suggested and a large number of arguments has been put forward for each of them. In previous work, we examined these proposals from a welfarist perspective [2] and assumed that the interests of the different states are uncorrelated. But this is too strong an idealization, as similar states have similar interests and therefore tend to cast the same votes. The new members of the EU are a case in point. They have similar problems and have to meet similar challenges; so a proposal that benefits, say, Poland will typically also benefit Slovenia; and proposals that harm Poland, will also harm Slovenia. There might also be negative correlations. For instance, a proposal which is good for the large states might be bad for the small states, and vice versa. This presence of correlations in the interests of states (and their corresponding voting behavior) raises the question if the decision rules that do best for uncorrelated interests will also do best if interstate correlations are taken into account. This is the question we will address in this paper.

The remainder of this paper is organized as follows: Section 2 introduces our framework and lays out some of the relevant mathematics. The following section 3 shows some of our results for vanishing correlations. Section 4 explains how correlations can be modeled in our framework. We introduce four different correlation patterns and run computer simulations. The results of these simulations are presented and discussed in section 5. The paper ends, in section 6, with some more general reflections.

---

<sup>1</sup>To make this principle more precise, an equality measure will be specified below.

## 2 Welfarism and social decision rules

The basic idea of the welfarist approach to social decision making is that, whether a proposal is accepted or rejected, makes a difference to the welfare of the people in the federation. Here is an illustration. Let us assume that it is proposed to construct a freeway in Portugal. If the proposal is rejected, nothing changes. No one can profit from the freeway, and no one has to pay for it. If the proposal is accepted, then the people of Portugal will, on average, gain utility (as they get, for example, faster to work every day) while the people in, say, Austria will, on average, lose utility as they have to contribute to its costs without having a chance to use it that often.

Whether a proposal is accepted or not depends on the decision rule. Weighted decision rules assign different weights to different states. Consider the freeway example again and assume that a weighted rule is adopted. Clearly, if the weights of Portugal are much larger than the weights of Austria, then the freeway proposal might get accepted with the result that the Portuguese can sleep longer in the morning and the Austrians are left with the bill. If, on the other hand, the weights of Austria are much larger than the weights of Portugal, then the situation might be the other way round. In the end, the challenge is to find a decision rule that leads to a good welfare distribution according to our principles.

But there is a challenge ahead: We do not know the proposals beforehand. To account for this uncertainty, we set up a probabilistic framework.

Let us now formalize these ideas. We consider a federation of  $m$  states with a total number of  $N$  people. States are numbered from 1 to  $m$  and labeled by lowercase letters (e.g.  $i, j$ ). The  $i^{\text{th}}$  state has  $N_i$  inhabitants. Of course,  $\sum_i N_i = N$ .

We model the proposals as exogeneous. A single proposal is represented by a utility vector  $\mathbf{v} = (v_1, \dots, v_m)$ . Here  $v_i$  is the average utility that people from state  $i$  will receive, if the proposal is accepted.<sup>2</sup>  $v_i$  is positive, if there is an average gain in utility for people from state  $i$ , and it is negative, if there is an average loss in utility for people from state  $i$ . The status quo is normalized to 0 and a rejected proposal leads to a zero average utility transfer. Since we do not know the proposals in advance, the utilities  $v_i$  are values of random variables  $V_i$  ( $i = 1, \dots, m$ ).

The vote of state  $i$  (or its representative) is described by another random variable  $\Lambda_i$  with values  $\lambda_i$ .  $\lambda_i = -1$  means that state  $i$  votes against the proposal, and  $\lambda_i = +1$  means state  $i$  votes for the proposal.  $(\lambda_1, \dots, \lambda_m)$  is a *voting profile*.

---

<sup>2</sup> *Average* utilities should not be confused with *expected* utilities which we will discuss below. Average utilities are means over people, expected utilities are means over different proposals that follow a particular probability distribution. Note also that we start with a rather coarse-grained description of decision making. A more fine-grained view would begin with the utilities of the individual people in the federation. Accordingly, we will only consider inequality at a coarse-grained level, i.e. on the level of states, and not of individual people.

How do states vote, if a certain proposal  $\mathbf{v}$  is on the table? We assume that each state examines the average utility that the proposal will confer to its own people. If the average utility is positive, it will vote in favour of the proposal. If the average utility is negative, it will vote against the proposal. In mathematical terms, the vote of state  $i$  is then given by  $\lambda_i = \text{sign}(v_i)$ .<sup>3</sup>

A *decision rule* can be represented as a function  $D$  from voting profiles  $(\lambda_1, \dots, \lambda_m)$  to  $\{0, 1\}$ . It takes the value 1, if the proposal is accepted, and the value 0, if the proposal is rejected.

Suppose now, that a decision rule has been adopted and that a particular proposal  $\mathbf{v}$  is on the table. How will the decision affect the average utilities for the different states? Call  $u_i$  the average utility that people from state  $i$  will receive from a decision on  $\mathbf{v}$ . According to our assumptions, we have:

$$u_i = v_i \times D(\lambda_1(v_1), \dots, \lambda_m(v_m)) . \quad (1)$$

Since the  $v_i$ s are values of random variables, so are the  $u_i$ s. We denote the corresponding random variables by  $U_i$  for  $i = 1, \dots, m$ .

The expectation values of these random variables,  $E[U_i]$ , are the key quantities in our welfarist framework. Once we know them, we can calculate other quantities that are required by our two principles. Utilitarianism requires the average expected utility of a person in the EU which is given by

$$E[U] = \frac{1}{N} \sum_i N_i E[U_i] . \quad (2)$$

Egalitarianism requires an equality measure. To keep things simple, we measure the spread in the distribution of the  $E[U_i]$ s. Let us call this measure  $I$ . If  $I$  is small, then the equality in the federation is high. If  $I$  is large, then the equality is low.<sup>4</sup>

Let us now calculate the expected utility  $E[U_i]$  for state  $i$ . To do so, we need the joint probability distribution  $p(\mathbf{v})$  over the proposed utilities. According to Eq. (1), we have

$$E[U_i] = \int d\mathbf{v} p(\mathbf{v}) v_i D(\lambda_1(v_1), \dots, \lambda_m(v_m)) , \quad (3)$$

where the integral over  $d\mathbf{v}$  is  $m$ -dimensional. Note that the decision rule  $D$  is a function of the voting profiles which are, in turn, a function of the  $v_i$ s.

For further analytical calculations, Eq. (3) can be rendered more manageable. To do so, we hold a voting profile  $(\lambda_1, \dots, \lambda_m)$  fixed. The probability that voting profile  $(\lambda_1, \dots, \lambda_m)$  occurs is  $p(\lambda_1, \dots, \lambda_m)$ . It is given by

$$p(\lambda_1, \dots, \lambda_m) = \int d\mathbf{v} \theta(\lambda_1 v_1) \dots \theta(\lambda_m v_m) . \quad (4)$$

<sup>3</sup>We need not consider the case of  $v_i = 0$  here, as it has zero measure under any reasonable probability distribution.

<sup>4</sup>We assume here that each person in state  $i$  receives the average utility  $E[U_i]$  and calculate the standard deviation of the expected utilities of single people. Note that this is nothing but a first quick-and-dirty estimate of the inequality in the federation. There are other measures, such as the Gini coefficient, that might be more appropriate.

Similarly, we calculate the expected utility of state  $i$  if the voting profile is  $(\lambda_1, \dots, \lambda_m)$ :

$$\bar{v}_i^{\lambda_1, \dots, \lambda_m} = \int d\mathbf{v} v_i \theta(\lambda_1 v_1) \dots \theta(\lambda_m v_m) / p(\lambda_1, \dots, \lambda_m). \quad (5)$$

With  $p(\lambda_1, \dots, \lambda_m)$  and  $\bar{v}_i^{\lambda_1, \dots, \lambda_m}$  we can now calculate the expected utility  $E[U_i]$  of state  $i$ :

$$E[U_i] = \sum_{\lambda_1, \dots, \lambda_m} \bar{v}_i^{\lambda_1, \dots, \lambda_m} \times p(\lambda_1, \dots, \lambda_m) D(\lambda_1, \dots, \lambda_m). \quad (6)$$

To simplify things a bit more, we assume that the marginals for the different states, i.e.

$$p_i(v_i) = \int dv_1 \dots \int dv_{i-1} \int dv_{i+1} \dots \int dv_m p(\mathbf{v}), \quad (7)$$

are identical. This means that, on the level of the proposals, there is no bias towards one or the other state. We furthermore assume that the marginals are normally distributed with a mean  $\mu$  and a standard deviation  $\sigma$ . All utilities are scaled such that  $\sigma = 1$ .<sup>5</sup>

### 3 Independent utilities from proposals

In order to explore the welfarist framework, we start with a simple *default model* in which the  $V_i$ s are independent. We will later relax this assumption. In the default model, the joint probability distribution  $p(\mathbf{v})$  factorizes:

$$p(\mathbf{v}) = p_i(v_i) \dots p_m(v_m). \quad (8)$$

This means that the utilities from proposals are uncorrelated for the various states. If one knows that a proposal puts benefits on the Fins, one cannot infer anything about the benefits or harms for people from other states. In order to refer to Eq. (8) more quickly, we will somehow loosely say that the states are independent. Note, however, that, even under Eq. (8), the random variables  $U_i$  are *not* independent, but correlated. The reason is that the decision takes all  $v_i$ s into account.

Under the assumption of Eq. (8), the sum in Eq. (6) can be worked out analytically or directly calculated by a computer program. For details, see [2].

To apply our methodology to the decision making in the European Union, we consider five decision rules that were discussed in the context of the constitutional reform of the EU.<sup>6</sup> These decision rules can be organized into two

<sup>5</sup>If the utilities are independent in the same state, we would expect, according to the central limit theorem, that the standard deviations for the different states are proportional to  $1/\sqrt{N_i}$ . However, [3] present a model with correlations within the same state that justifies our choice of identical standard deviations.

<sup>6</sup>For a complementary approach in terms of expected utility see [4].

groups. In the first group are three *theoretical rules* that assign a weight  $w_i$  proportional to  $N_i^\alpha$  with  $0 \leq \alpha \leq 1$  ([5]) to each state  $i$  (see [6], Chapter 2). The weights are normalized to 1, i.e.  $\sum_i w_i = 1$  and a proposal is accepted if the combined weights of the states which vote for the proposal exceeds a threshold of .5. We consider the following theoretical rules.

**(SME)** Simple majority with equal weights ( $\alpha = 0$ ).

**(P50)** Simple majority with square root weights ( $\alpha = .5$ ).<sup>7</sup>

**(SME)** Simple majority with proportional weights ( $\alpha = 1$ ).

In the second group are two *political rules*, which are more complex than the theoretical rules. Here each state is assigned several weights, which are aggregated separately. A proposal is accepted, if the aggregates exceed their respective thresholds (for details see [2], Section 2).

**(Acc)** This rule, which is formulated in the Accession Treaty and which builds on the Nice Treaty, is presently in force. It identifies three classes of weights, one with  $\alpha = 0$  (threshold 50%), one with  $\alpha = 1$  (62%), and one with an unsystematic weights (72%).

**(Con)** This rule is part of the Constitution that is presently in the process of ratification. It identifies two classes of weights, one with  $\alpha = 0$  (threshold 58%) and one with  $\alpha = 1$  (65%).

Let us now briefly consider results for the default model (for details, see [2]). In Figure 1 we show the expected utility of an average person in the EU (left panel) and our measure of inequality (right panel). The larger the spread, the more inequality we find in the federation. Our characteristics are shown as a function of  $\mu$ , the mean over the utilities from proposals.

Let us first consider expected utility. For  $\mu$  significantly smaller than 0, proposals are typically bad. They are therefore mostly rejected, and the utilities of the people in the federation do not change. A closer inspection of the curves shows that the political rules do slightly better for a range of negative  $\mu$ -values.

For  $\mu$  significantly larger than 0, the proposals are typically very good. Therefore, most of them are accepted under any decision rule. As the utilities are now conferred to the people,  $E[U]$  will be positive. For  $\mu > 1$ , the curves for the different rules almost coincide.

The most interesting range is the one around  $\mu = 0$ . This is also the most realistic range of parameters, as we argue in sec. 5 of [2]. In this range the decision rules yield significantly different results. The general trend is that the theoretical rules do better. At  $\mu = 0$ , SMP is the best rule, followed by P50 and SME.

Let us now turn to equality. As the right panel of Figure 1 shows, SMP does very badly in terms of equality for  $\mu \approx 0$ . It is followed by P50 and the political rules. SME exactly equalizes the expected utilities for any value of  $\mu$ .

<sup>7</sup>This rule is named after Penrose, who invented it. See [8].

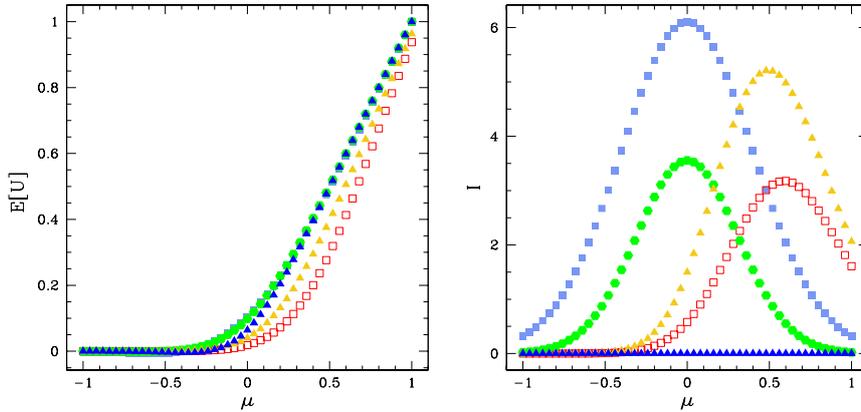


Figure 1: The expected utility (left panel) and the measure of inequality (right panel) as a function of  $\mu$  for the alternative decision rules. Different point styles designate different rules. Filled light blue squares: SMP; filled green circles: P50 (square root weights); Filled dark blue triangles: SME. Red open squares: Acc. Filled orange triangles: Constitution.

So far we have ranked a few decision rules. But there is still the question, whether we have found the best rules on our desiderata. For the default model, there are a few analytic results in this respect. [4] specify the best decision rule in terms of expected utility – expected utility is maximized under proportional weights and a threshold that depends on  $\mu$ . An alternative proof for this result is given by [3]. [3] also provide analytical arguments regarding the egalitarian desideratum. They are based upon a relation to Banzhaf voting power (see [6]).

## 4 Modeling correlations

So far, our results assume that the utilities from proposals are uncorrelated for the different states. But this assumption is not realistic, as we have argued above. Thus the question arises whether the results we obtained for the default model are stable if correlations are taken into account. To address this question, we concentrate on the case of  $\mu = 0$ .

To model correlations between the states, we assume that  $p(\mathbf{v})$  is a multivariate normal. It is fully determined by its covariance matrix. The entries in this matrix are  $c_{ij} = E[V_i V_j] - E[V_i]E[V_j]$ , where one has to take the expectation value over the probability distribution  $p$  in order to calculate  $E[\cdot]$ .  $c_{ii}$  is the variance for the utility from proposals for state  $i$ . We assume that it is set to 1 for  $i = 1, \dots, m$  as before.

As there is a lot of freedom to specify the entries  $c_{ij}$  and as we are interested in typical behavior that arises from correlations amongst the states, we

CP	type	neg. cross corr.	$\alpha = 0$	$\alpha = .5$	$\alpha = 1$
1	small/large	no	.84/.16	.64/.36	.43/.57
2	South/North	no	.48/.52	.49/.51	.48/.52
3	small/large	yes	.84/.16	.64/.36	.43/.57
4	South/North	yes	.48/.52	.49/.51	.48/.52

Table 1: The parameters used in patterns CP1 to CP4. The numbers in the  $\alpha$ -columns are the aggregated weights of the states in each group.

define four *correlation patterns* (CP1 – CP4). Each correlation pattern has one parameter ( $\varrho$ ) which measures the strength of the correlations. In the covariance matrix, every off-diagonal entry is scaled by  $\varrho$ .  $\varrho = 0$  means vanishing correlations.

Each correlation pattern groups the states of the EU into two groups of similar (population) size. Patterns CP1 and CP3 consider larger vs. smaller states, and patterns CP2 and CP3 southern vs. northern states (see Table 1 for details).

**CP1–2** States  $i, j$  from the same group are correlated with strength  $c_{ij} = \varrho$ . States  $i, j$  from different groups are uncorrelated ( $c_{ij} = 0$ ).

**CP3–4** States  $i, j$  from the same group are correlated with strength  $c_{ij} = \varrho$ . States  $i, j$  from different groups are negatively correlated with  $c_{ij} = -\varrho$  ( $\varrho > 0$ ) reflecting the “zero-sum” character of (at least) some of the decision making progresses in the EU: The gains of one states equal the losses of another state.

While the case of zero correlations could be dealt with analytically, the case of non-zero correlations requires the use of computer simulations. They are done as follows. We evaluate the integral Eq. (3) in a Monte Carlo way. As many Monte Carlo integrations, our simulations allow for a dynamical interpretation in terms of an intuitive picture. The picture is as follows: We randomly draw utilities  $v_i$  according to our multivariate normal. We determine the votes of the states and check whether the proposal is accepted or rejected. If it is accepted, the respective utilities are distributed to the states, if not, nothing changes. We repeat this  $N_{sim} = 10^6$  times. In practice, the procedure converges quickly. In order to get fast random numbers following a multivariate normal, we make a coordinate transformation so that the correlation matrix becomes diagonal.

## 5 Results

Let us now turn to the outcomes of our simulations which are depicted in Figs. 2 to Fig. 5. The figures exhibit a rich structure and we will restrict ourselves to a discussion of the main results.

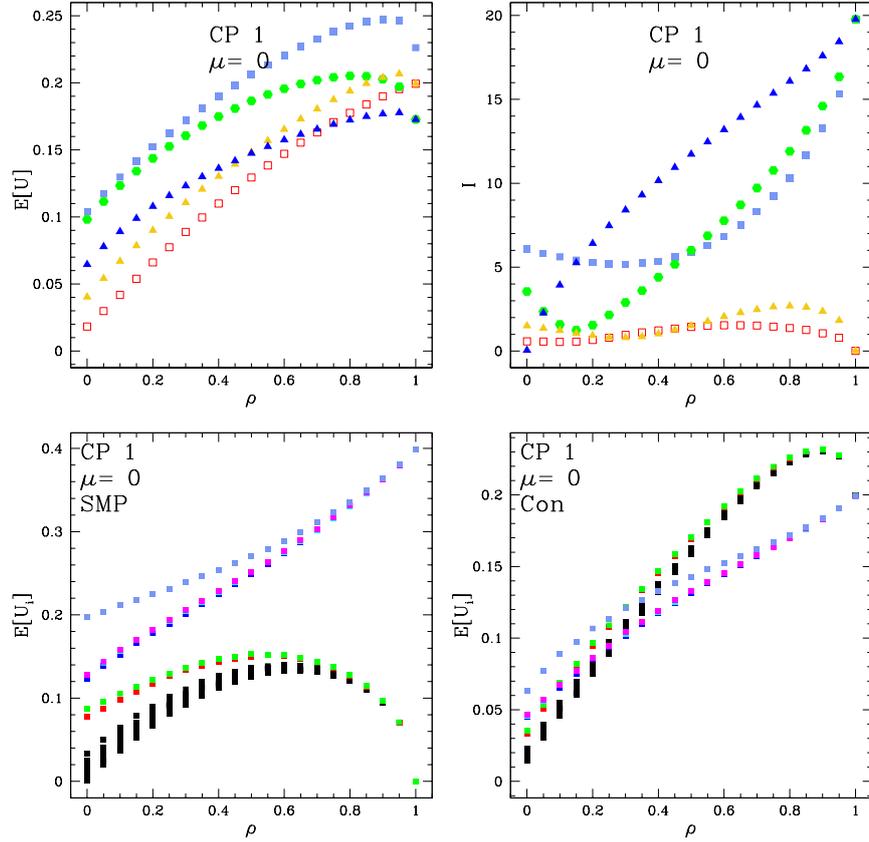


Figure 2: Different characteristics as a function of  $\rho$  for CP1. Left top panel: expected utility  $E[U]$ . Right top panel: variance  $\sigma_w(\{U_i\})$ . Point styles in the top panels as in Fig. 1. In the bottom panels we consider one rule and show the expected utilities  $E[U_i]$  for every state  $i$ . Left bottom panel: SMP. Right bottom panel: Constitution. The point styles are different here: Poland (red), Spain (green), Italy (dark blue), U.K. (cyan), France (magenta), Germany (light blue), all other states (black).

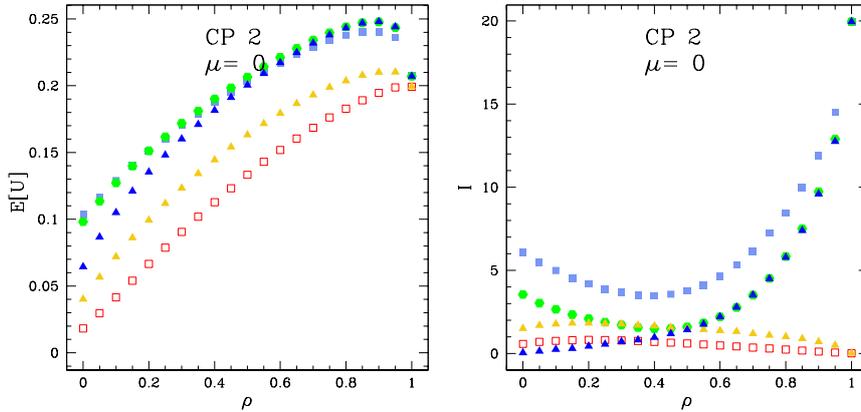


Figure 3: Results for CP2. Point styles in the top panels are as in Fig. 1; point styles in the bottom panels are as in the bottom panels of Fig. 2.

We reserve one figure for each correlation pattern. We show the expected utility  $E[U]$  and the standard deviation of  $E[U_i]$ s as a function of the correlation strength  $\rho$  for our rules (Fig. 2 contains two more panels, which we will consider presently). The leftmost point ( $\rho = 0$ ) corresponds to the point  $\mu = 0$  in Fig. 1. Note that the ranking changes whenever two lines intersect.

The most important question is: Does the ranking of the various decision rules that we obtained for the default model (Fig. 1) change if correlations are taken into account? As Figs. 2 to 5 show, this ranking is fairly stable, as far as the expected utility is concerned. Regarding inequality, there is one significant change: SME, which minimizes inequality under the default model, is worse than the political rules for all correlation patterns and a large range of correlation strengths  $\rho$ . Apart from this, the political rules are better in terms of equality than SMP and P50 both under the default model and if correlations are turned on.

Let us now look at the expected utility of the whole federation,  $E[U]$ , in more detail and explain some of its features. Whereas, under CP1 and CP2, the expected utilities tend to increase with increasing correlation strength, they decrease under CP3 and CP4. The reason is as follows: The most significant contribution to  $E[U]$  comes from proposals from which people from many states benefit. Under CP1 and CP2, there are only positive correlations. The stronger these correlations are, the more likely proposals will benefit people from many states in the federation. Thus,  $E[U]$  increases as a function of the correlation strength. This holds quite independently of the respective decision rule. Note, however, that, around  $\rho \approx .9$ , things get more complicated, and particularly SME is outrun by the political rules.

Under CP3 and CP4, on the contrary, there are more negative correlations than positive correlations. So typically, if people from one group of states re-

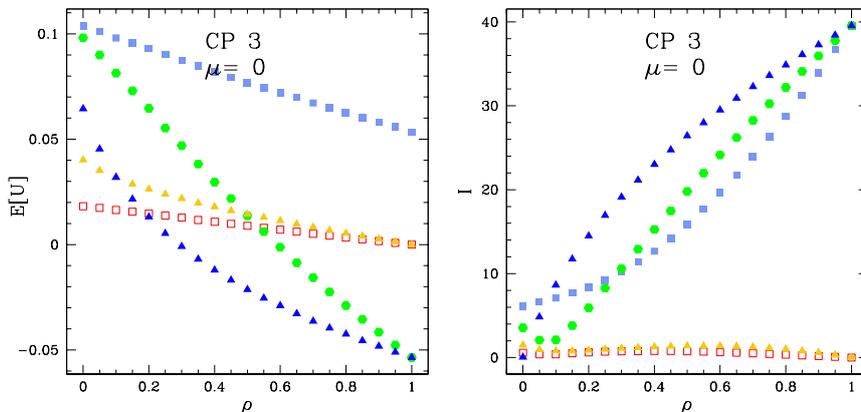


Figure 4: Results for CP3. Point styles as in Fig. 1.

ceive benefits, people from the other group have to pay. Accordingly, proposals from which people from many states take profits, become less likely, and  $E[U]$  decreases, as  $\varrho$  increases.

The curves for CP3 are very peculiar. SME and P50, which do reasonably well under the default model, are outrun by the political rules for  $\varrho \gtrsim .2$  and  $\varrho \gtrsim .5$ , respectively. At  $\varrho = 1$ , Acc and Con result in zero expected utility for the federation, whereas SME and P50 produce a negative expected utility. Here is an explanation for this behavior. Under CP3, the large states have the majority of people. However, as Table 1 shows, under SME and P50, the small states hold more weights than the threshold requires. At sufficiently large values of  $\varrho$ , the small states are very likely to vote in the same way. Thus, if a proposal is accepted, it will very likely benefit most of the small states. However, because of the anticorrelations in CP3, such a proposal tends to be harmful to people from the larger states. And since there are more people from larger states than from smaller states,  $E[U]$  will drop below zero.

The political rules, on the other hand, have higher thresholds of acceptance. A proposal is only accepted, if both large and small states vote for it. As  $\varrho$  increases, under CP3, proposals will less likely put benefits on both people from large and from small states. Accordingly, large and small states are less likely to cast the same vote “yes”. As a result, proposals are less likely to be accepted, and  $E[U]$  approaches 0.

The lesson is, clearly, as follows: If there are two groups that have anticorrelated interests, it is very bad in terms of expected utility to give the smaller group more weights than the threshold requires.

Let’s now look at our measure of inequality  $I$  in more detail (right panels). Overall, the curves look very similar: As  $\varrho$  increases, the measure of inequality for the theoretical rules increases. At  $\varrho = 1$ , a maximum value of  $I$  is reached. The political rules change a bit in terms of  $I$  and approach  $I = 0$  at  $\varrho = 1$ .

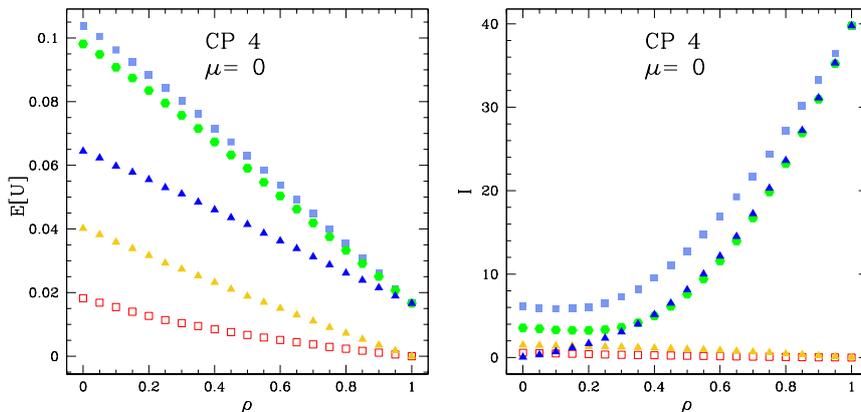


Figure 5: Results for CP4. Point styles as in Fig. 2.

The explanation can be obtained from the bottom panels of Fig. 2, where the  $E[U_i]$ s are shown as functions of  $\varrho$ . We observe that two groups are formed in the following sense: As  $\varrho$  increases, the  $E[U_i]$ -values of states from the same group get closer. However, whereas, under Con, the expected utilities for the groups converge in the limit of  $\varrho = 1$ , they diverge under SMP. This produces a finite variance. The explanation should come as no surprise, given what we have said before. Under the political rules, states from both groups are needed for acceptance. As a consequence, it will not make a big difference in terms of  $E[U_i]$ , to which group a state belongs. At  $\varrho = 1$ , the only proposals that have non-zero probability and that yield acceptance, put the same utility to people from every state. So there is zero variance. Under the theoretical rules, on the other hand, the states from one group will hold more weights than the threshold requires. Accordingly, it makes a big difference to  $E[U_i]$ , whether state  $i$  is a member of this group or not, and the variance approaches a finite value.

Note, that the ranking of the theoretical rules is different for the different correlation patterns, as far as  $I$  is concerned. For CP1 and CP3, SME is worst for a large range of  $\varrho$ -values, whereas SMP does worst for almost all value of  $\varrho$ .

We also obtained results for finite  $\mu$ -values. Overall, our results do not change much, as we move to finite  $\mu$ -values of the order of .2 (other values of  $\mu$  are not realistic).

Again there is the question, whether we have found the best decision rules on our desiderata. From the proof of Theorem 1 in [4] one can construct the rule that maximizes expected utility, even if there are non-zero correlations. Unfortunately, this decision rule is very complicated in general and not suitable for practical purposes. So we think it more appropriate to start with some subset of simple decision rules and to look for the best of them, as we did. But for this it is certainly useful to scan the range of  $\alpha$ -values more systematically. We leave that for future work. Regarding equality, we are not aware of analytic

arguments on the best decision rule.

## 6 Conclusions

The welfarist framework presented in this paper complements the axiomatic approach that has been dominating social choice theory for the last fifty years. We start from sensible desiderata that specify when one rule is better than another one and rank alternative decision rules with respect to these desiderata. Our approach allows us to naturally include “empirical” constraints (such as correlations in the interests of states). Our work profits from the rapid progress in computer science; this helps us to simulate proposals and votes that follow complicated probability distributions. The computational methods we adopt have been used in other disciplines, and we hope to have convinced the reader that they have much to offer to social choice theory as well.

In this paper, we found two main results. *First*, regarding expected utility, we obtain a fairly stable ranking of the decision rules, where SMP does best and the political rules do worst. This is suggested by our simulations of four different correlation patterns with varying correlation strength. We take the stability of the ranking to be good news for the welfarist framework – if the the ranking of the rules were too sensitive to the correlation pattern and the correlation strength, our account would be useless for policy recommendations. *Second*, the two welfarist principles that we studied in this paper, utilitarianism and egalitarianism, pull in different directions. Whereas political rules with high acceptance thresholds tend to do better in maximizing the expected utility of the federation, theoretical rules are superior in achieving equality. As both principles cannot be satisfied at the same time (at least by the rules studied in this paper), one has to strike a compromise. For vanishing correlations, the rule SME seems to be a reasonable candidate: It yields no inequality at all and is at least better than the political rules in terms of expected utility. Unfortunately, this result does not hold anymore for finite correlations, where SME may produce inequalities that are much larger than the inequalities under political rules.

Another way to compromise between utilitarianism and egalitarianism is to introduce relative weights for these principles. We leave this for future research. We also plan to find realistic correlation models that adequately reflect the correlations of votes found in empirical data.

## References

- [1] K. J. Arrow. *Social Choice and Individual Values* (2nd edition). Wiley, 1963.

- [2] C. Beisbart, L. Bovens and S. Hartmann. A utilitarian assessment of alternative decision rules in the Council of Ministers. *European Union Politics*, 6(4): 395–419 (2005).
- [3] C. Beisbart and L. Bovens. Welfarist evaluations of decision rules for boards of Representatives. Working paper, 2006
- [4] S. Barberà and M. O. Jackson. On the weights of nations. *Journal of Political Economy*, 114(2): 317–339 (2006).
- [5] L. Bovens and S. Hartmann. Welfare, voting and the constitution of a federal assembly. Forthcoming in: M.C. Galavotti, R. Scazzieri and P. Suppes (eds.), *Reasoning, Rationality and Probability*. CSLI Publications, 2006.
- [6] D. S. Felsenthal and M. Machover. *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*. Edward Elgar, 1998.
- [7] D. S. Felsenthal and M. Machover. Enlargement of the EU and weighted voting in its Council of Ministers. URL: <http://eprints.lse.ac.uk/archive/00000407/01/euenbook.pdf> (2000)
- [8] L. S. Penrose. The elementary statistics of majority voting, *Journal of the Royal Statistical Society* 109: 53–57 (1946).
- [9] A. Sen. *Collective Choice and Social Welfare* (2nd edition) Holden-Day, 1984.

Claus Beisbart  
 Institute for Philosophy, Faculty 14  
 University of Dortmund  
 D-44221 Dortmund, Germany Email: [Claus.Beisbart@udo.edu](mailto:Claus.Beisbart@udo.edu)

Stephan Hartmann  
 Department of Philosophy, Logic and Scientific Method  
 London School of Economics and Political Science  
 Houghton Street  
 London WC2A 2AE, UK Email: [S.Hartmann@lse.ac.uk](mailto:S.Hartmann@lse.ac.uk)