

Mean, Mode and Median Utilitarianism

Jonathan Wolff

Dept of Philosophy

UCL

Average utilitarianism is rarely discussed in its own right. Although Rawls remarks that the moral underpinnings for average and classical utilitarianism are quite distinct, in general the two doctrines will recommend the same outcome: whatever maximises total happiness will maximise the average. As is well-known, the exception to this is in population change cases, where we might increase average happiness by measures that decrease total happiness; by executing the relatively miserable for example. On the other hand we might increase the total by measures that reduce the average, by bringing into existence people who have the most modest degree of happiness in their lives.

These considerations may explain why average utilitarianism is something of a problem theory. Either it collapses into classical utilitarianism, or, where it doesn't, it raises serious problems for consequentialism. Hence consequentialists might prefer to leave it in the dark. But we have our first two problems for average utilitarianism: in non-population change cases it collapses into classical utilitarianism, and in population change cases it can recommend slimming the population down, ultimately to the ecstatic.

Given that in standard cases average utilitarianism is identical in its recommendations to classical utilitarianism, it is no surprise that it will also inherit some of the problematic features of the latter. First (or rather, third, given that we already have the problem of collapse and the problem of the ecstatic) any maximising doctrine needs quantification, and in this case we need inter-personal cardinal comparisons of utility. Despite various claims to have solved this problem, nothing, as far as I know, has yet to convince anyone save its inventor. This clearly, is a challenge for average utilitarianism. Fourth, as Nozick has argued, we might be able to maximise utility by giving everything to those who are most efficient at converting resources into utility: the problem of utility monsters, strange beings who get increasing marginal returns from goods. This afflicts both average and classical utilitarianism, of course. Finally, there is the notorious problem of victimisation, or the sacrifice of some for the sake of the greater happiness of a greater number. This differs from the problem of the ecstatic in that the latter instructs us simply to eliminate the less happy to raise the average. The problem of victimisation tells us to engage in policies that lead to a greater total balance of pleasure over pain, even if the benefits and burdens fall very unequally: perhaps even requiring the torture of some for the benefit of many others.

So average utilitarianism has two problems of its own, and three it inherits from classical utilitarianism. Of course it may have many more, but this is enough to be going on with.

Now, so far in all of this we have identified the notion of 'average' with 'mean', which is, of course, the most usual understanding, and, as far as I know, the universal reading of average utilitarianism. Yet from primary school mathematics we know that there are several different common notions of average. In addition to the mean, there is the median and the mode. The median is 'the one in the middle'. That is, if you lay out the sample set in order from greatest to smallest, the median is the one in the middle of the array. The mode is, essentially, 'the most common'. Of the sample set, the mode is the value that is held by more of the sample than any other value. If we were to draw a bar graph, with values at the bottom (the x axis), and quantities on the y axis, the mode would be the highest bar. So suppose we have a sample of the following sort:

1 unit of value:	3 samples
2	2
3	2
4	5
5	6
6	1
Total value	64
Total samples	19
Mean	3.36
Median	4
Mode	5

It is not uncommon for the mean, median and mode to coincide, and, indeed, it is sometimes said that an average is only statistically significant if they do. However we can see from this example that they can diverge.

Now as far as I know average utilitarianism has always been assumed to be 'mean utilitarianism'. I have not looked very hard, but I have not found a discussion of the possibility of median or mode utilitarianism. This is somewhat surprising as utilitarianism has often seemed attractive to those on the margins of philosophy and economics, who would tend to be more aware of elementary

statistics than philosophers. But be that as it may, it seems worth taking a look to see whether any of the problems laid out so far can be answered by changing the understanding of average utilitarianism.

Median Utilitarianism

Median utilitarianism, of course, tells us to maximise the median happiness of the relevant population. This means that we should choose the distribution which has the happiest 'middle person': the person in the middle of the array. So in a population of 99, the 'median person' is the person who is not less happy than 49 others and also not more happy than 49 others (this awkward formulation reflects the point that there may be more than one person at the same happiness level). Possible distributions can be assessed on the basis of how happy the person in that position would be. (Of course the median person in one possible distribution may not be the median person in another.)

How plausible is this position? Intuitively there may be something to be said for it. One oft-noted problem with using the mean is that it often seems abstract to the point of absurdity. We are told that the average family used to have 2.4 children, but now it has dropped to 1.9. While we know what this means, it also has a somewhat comical. Alternatively, we might designate a particular, really existing, family 'the average family' and count the number of children they have, to see how many children there are in the average family. This is much closer to the notion of the median. Consequently if we think that the point of utilitarianism is not to maximise the average happiness, but to make the average person as happy as possible then we are thinking in median terms rather than mean terms. Technically there seems to be little bar to defining a position we can call 'median utilitarianism'.

However this is not to say that the notion of median utilitarianism is unproblematic. First I shall run through the five problems outlined for mean utilitarianism to see whether median utilitarianism manages to avoid them, and then consider where this leaves us.

The first problem was that of collapse. Mean utilitarianism recommends the same policies as classical utilitarianism, as long as population changes are not in issue. Clearly, however, this will not be the case with median utilitarianism. You can change the total happiness without affecting the median, by raising or lowering the happiness of those above or below the median, provided the median person is unaffected. Similarly redistribution may allow for changes to the median without changing the total. Collapse is no longer an issue.

The problem of the ecstatic was that mean utilitarianism appears to recommend 'eliminating' all those of below mean happiness, simply in order to raise the mean. Indeed this process reiterates until all one is left with is a small group of ecstatically happy people. Could the same happen to raise the

median? It seems that median utilitarianism has no advantage here. Eliminating those at the bottom of the distribution changes the median, and, indeed, will raise it. This stubborn problem for consequentialism cannot be avoided by adopting median utilitarianism.

The next problem was one of quantification. Mean utilitarianism requires full, cardinal, interpersonal comparisons. The informational demands of median utilitarianism are significantly less. Although we need to be able to put all members of the population in an ordering, in order to determine the median, and to compare one possible median position with another, in order to see which is to be preferred, this requires only ordinal inter-personal comparisons. We do not need to quantify happiness; rather we need to make judgements about whether one person is happier than another. Now although there may be very serious questions about how we do this, it is a far less daunting task than full, cardinal, comparability. Thus the problem of quantification is greatly reduced.

Utility monsters are our next concern. Suppose we have a person who is such an efficient converter of resources into utility that we maximise utility by giving that person everything. What effect would this have on the median? Most likely it would lower it. But even if the gains were all made by transferring resources from those below the median to the utility monster, the resulting increase in total utility would not affect the median. Therefore median utilitarianism sees no reason to transfer resources to utility monsters.

The obvious attempt to reply is to point out that if the median person is herself a utility monster then the objection re-occurs. However, we have to remember that 'median person' is a non-rigid designator and to increase the position of the median person we may have to make all of the top 50% of the population better off. Now we could, I suppose, imagine that the top 50% were all utility monsters, and this would mandate transfers from the remainder. However, with these figures it seems we are no longer talking about the problem of utility monsters, but the problem of victimisation. This is the point that we may be required by the theory to engage in policies which benefit some at the cost of others, including others who may already number among the worst off, or would become so. Once more I think we have to concede that in principle victimisation could be required under median utilitarianism. Indeed, it could happen in a particularly unfortunate way. For the theory requires us to concentrate on the median position which, we have just seen, means maximising the happiness of the person at, roughly, the 50th percentile. But what about the person at the 51st percentile? They can go hang! So what we can expect is a huge gap between the median person and the next one in the array. Indeed any improvement in the position of the median person, however trivial, requires the sacrifice of anyone below the median, however significant. So in this respect median utilitarianism is in a far worse position than mean utilitarianism. For although lowering the position of those below the mean affects the mean,

lowering the position of those below the median does not affect the median. Consequently mean utilitarianism will outlaw some forms of victimisation that median utilitarianism requires.

In sum, although median utilitarianism has its advantages - - it doesn't collapse into classical utilitarianism; it doesn't require cardinal inter-personal comparisons of utility; and it doesn't suffer the problem of utility monsters - - it is deeply flawed. Indeed its flaws and some of its advantages have the same source: its utter indifference to everything except the plight of the median person. This is a reason why utility monsters don't get more, but, at the same time, why it requires us to engage in rampant victimisation of those below the median. It is not, in the end, an improvement. It deserves its neglect.

Mode Utilitarianism

This leads us to the next candidate, mode utilitarianism. The mode, we saw, is essentially, 'the most common'. I said above: 'Of the sample set, the mode is the value that is held by more of the sample than any other value'. In graphical terms, the mode is the highest bar on the chart.

Can we even formulate 'mode utilitarianism'? The immediate obstacle is that the notion of the mode seems to be clear only when the values involved are discrete, rather than continuous. Values are continuous if, for any two distinct values, there is always a third intermediate between them. To put the problem simply; the mode is well-defined only when you can draw a bar graph. But with continuous values you cannot draw such a graph. To illustrate, it makes perfect sense to ask what the mode number of children is, in households in the UK. As the possible values are: 0, 1, 2, 3, 4 etc, it makes perfect sense to ask what the mode is. Here we can draw a bar graph. The answer is determinate: I would guess 0. But suppose we are interested in the height of the British population. We can understand talk of the mean, and the median, but what could be meant by the mode height? After all, if our measuring instruments are accurate enough it may even be that there are no two people with exactly the same height. The technical problem is not that there are too many possible values, although even this may make talk of the mode uninteresting. (For example, the number of hairs on a person's head is a discrete quantity, and there must be an answer to the question of what the mode number of hairs is among (non-bald) Londoners. We could, in principle, draw a bar graph. And I have to confess that I would be quite interested in seeing it. But although the distribution might be interesting, it is likely that the mode would not be, as any number is unlikely to be shared by many people.) Rather the problem is that there is no limit to the number of values. Without further work, no bar graph can be drawn. Consequently mode utilitarianism would only seem to make sense if happiness is a discrete commodity. But it seems obviously continuous. Thus there is no immediately clear way of settling the question of the mode.

Is the game up? Possibly, but perhaps it is worth trying a bit harder. After all, Bentham's slogan 'the greatest happiness of the greatest number' could be taken as a statement of mode utilitarianism. We take the biggest group and make them as happy as possible. What can we do at least to state a version of this to consider?

The analogy between happiness and height is worth pursuing. Suppose we wanted to increase the mode height. What is the mode? As height is a continuous quantity we run into the problem set out above. What can we do? One possibility is to use semi-arbitrary bands. We could place people in groups sorted into groups. In practice this is what clothes manufacturers do. So, to change the example slightly, consider women's dress size. Women consider themselves to have a size, which seems to start at 8 (below that are children's sizes), and then proceed by even numbers into increasingly high numbers. This banding provides a surrogate discreet quantity, and given that, the notion of the mode dress size makes perfect sense. We can draw our bar graph. When Marks and Spencer report that women's sizes are changing, and, according to their surveys the average woman's size is now a 16, it is, at first, unclear whether they are taking this to be the mean, mode, or median. It is very unlikely to be the mean, for it would be a great coincidence for it to be a whole number (It could, of course, be the mean rounded up or down.). But the median would be of relatively little interest to the retail trade. It is only by knowing the mode that they will know what to put on their shelves. So I suspect that by saying that the average woman is now size 16 they mean that there are more women of size 16 than of any other size. So settling upon the mode, in cases of size, can be tractable.

Nevertheless, we don't merely need to find the mode. Mode utilitarianism needs to maximise it, and this is another step. Pursuing the analogy with size will give us some bizarre policy statements, but let us press on regardless. Now we know that the mode dress size is 16, what would it mean to maximise the mode? The instruction is ambiguous. It could mean that we should make the mode size as high as possible - - now we have got to 16, let's aim for 18. Or it could mean that we should make as many people as possible members of the mode - - we should try to transform all those 14s and 18s to 16. Possibly it could mean a combination: make the mode as high as possible and as many people as possible members of the mode. But of course we may need a priority rule if we find we cannot do both.

Nevertheless, this gives us one strategy for understanding how to be a mode utilitarian. We put people into rough bands of happiness. Perhaps at one end we have the utterly despondent, then the chronically miserable, then the anxious, then the contented, and so on through the cheerful and the joyful, until we find ourselves with our old friends the ecstatic. Maximising the mode would require us to make the mode level of happiness as high as possible. If in world one, the mode group is the anxious, and in world two the mode group is the contented, we choose world two over world one. Once that is

done we try to move as many people as we can into the mode group. (Presumably we favour moving people from below the mode into the mode, although this will need examination). The distinctive feature of this analysis is that we band people according to happiness, putting them into a relatively small number of groups, and then find the possible distribution in which the mode group is the happiest.

However, this is not the only model we can use for interpreting mode utilitarianism. Note that the banding we propose is a banding in terms of degrees of happiness. But there is no reason why the banding couldn't be done in some other way. In the abstract, the idea would be to band according to one criteria, but to judge (and maximise) according to another. To illustrate, consider again what we considered the bizarre policy of trying to maximise the mode size. Dress size allows us to determine the mode size. But there is nothing at all bizarre about trying to make that group as happy as possible. The policy is not to get the mode size as high as possible, or even to get as many people into the mode as possible, but to take the mode group and make them as happy as we can. For want of better terminology we might make a distinction between 'reflexive mode utilitarianism' and 'irreflexive mode utilitarianism'. The former bands in terms of happiness and then tries to maximise the happiness of the mode; the latter bands in other terms, and then tries to maximise the mode.

Reflexive mode utilitarianism can be represented by ordinary bar graphs. Irreflexive mode utilitarianism needs something more sophisticated: a third dimension to the graph. The height of the bar identifies the mode, whereas its depth measures the happiness of the mode group. Thus first we find the mode group and second observe the thickness of its bar. We choose the possible world in which the thickness of the bar of the mode group is the greatest.

How might we flesh out an example of irreflexive mode utilitarianism? Although the policy of maximising the happiness of the mode dress size makes a lot of sense if you want to sell dresses, as a broader social policy it seems rather suspect. So we need to think if there is a more natural way of banding people for social policy purposes, which could be incorporated by a mode utilitarian. An obvious trial suggestion is the type of social/economic class banding that market researchers used to use. Thus the population is divided up into grounds A, B, C1, C2, D and E. We find the largest group, and try to make that group as happy as possible. Now there are three immediate questions to answer.

First, why that banding in particular? Now anyone advocating mode utilitarianism would need to justify their banding. But we are a long way from advocacy at this point, so I will let this question hang. We need to answer other questions.

Second, once we have found the largest group - - say C2 - - should we not only try to raise its happiness, but make the group as large as possible? I think we can simply reply that we need to make a

further distinction here between those approaches that try to shift people from one band to another and those that do not. But this is one cul-de-sac we will not go down.

Finally, and perhaps most seriously, we now have the instruction to make the C2 group as happy as possible. But what does that mean? Should we make the worst off C2 as happy as possible? That doesn't seem right. The average member? But average in what sense? Our problem begins to reiterate. Should we start banding within the bands? And banding within the bands within the bands? Or simply use the mean or median of the band? If we do, it looks like we will start collecting the objections that can be levelled against mean and median utilitarianism in addition to any fresh objections that mode utilitarianism might attract. So irreflexive mode utilitarianism looking more and more like a waste of time.

Still, we have reflexive mode utilitarianism - - where we band in terms of happiness - - and this does not fall prey to the problem just stated. For the idea is that we want to set up society such that the band with the highest number of people in is as high up the scale as possible. This does not require a further decision about what it means to maximise the happiness of people in that band, for they are already banded according to the happiness group in which we fall.

So we could settle for reflexive mode utilitarianism as the best statement of a mode utilitarian position. Yet we have another possibility: combining the two. What we might do is to provide our original banding in terms of social class (say). This tells us to concentrate on the C2s. Our problem was: what does it mean to maximise the happiness of the C2s, and the worry was that if we said we wanted to maximise the mode happiness of the C2s we are stuck in some sort of regress. But we can now see the way out. This second banding is to be done in a reflexive way. To explain, within the C2s we band by happiness, from the utterly despondent to the ecstatic, and our goal is to have the mode within the C2s as high as possible. This ends the regress. Again we can explain it with the dress size analogy. We want to make the size 16s as happy as possible. This means if we have a choice between a world in which the largest size 16 group is dejected and a world in which the largest size 16 group is contented, we choose the latter. So this combination of reflexive and non-reflexive mode utilitarianism does yield a coherent, if complicated, position.

So we now have two, apparently coherent, formulations of mode utilitarianism. Pure reflexive mode utilitarianism tells us to band in terms of happiness, and choose the world in which the biggest group is the happiest. Hybrid mode utilitarianism tells us first, to band on some other basis (say social class) second, assess the degree of happiness of that group by sub-banding in terms of happiness and finding the mode, and finally choosing whichever possible world has the happiest mode (i.e. the happiest biggest sub-band). To illustrate, we find, first, that C2s are the biggest group. We find that the

biggest sub-ground (banded by happiness) are the contented. In all other possible worlds the biggest sub-group is the anxious, so we conclude that maximising the mode happiness of the C1s requires us to stay where we are.

As I said, we now have two mode utilitarian theories. Are either of them remotely plausible? Let us assess this by running them against the problems set out before.

First, neither of them collapses into classical utilitarianism. This can be seen by reflecting on the fact that a policy may increase the mode and at the same time decrease the total. For example a policy may reduce the happiness of the very happy by a greater amount than it raises the happiness of a similar number of others, but may nevertheless raise the mode. The simple case is where it allows a number of people to shift from a lower group, the current mode, to a higher group which then becomes the mode. Thus total individual losses, when totalled, may outweigh total individual gains, yet the mode may rise. The problem of collapse is avoided.

The problem of the ecstatic was that, in order to increase the mean and median we can destroy those at the bottom of the distribution. This process reiterates until we are left just with the ecstatic. At first sight it may seem that mode utilitarianism avoids this, as destroying those at the bottom of the distribution will have no effect on the mode (unless the bottom of the distribution *is* the mode). However on a second look the victory is short-lived. For we realise that if we destroy everyone except those in the top band (or even those in the top sub-band) we increase the happiness of the mode - - by changing which group is the mode! So all forms of average utilitarianism suffer from this problem. It seems likely that no pure form of average consequentialism can avoid it.

The next problem was quantification. Mean utilitarianism requires full cardinal interpersonal comparisons of utility. Median required only ordinal interpersonal comparisons. This is also true, I think, of mode utilitarianism, on both understandings. Whether we decide to make the biggest social group as happy as possible, or to move the mode up as high up the bands as possible, we only need interpersonal ordinal comparisons of happiness. We also only need ordinal comparisons of group size, of course, but we can have even cardinal comparisons for free. However there is a problem of quantification in that we need to be able to define bands of happiness, on both understandings, and to be able to assign people to bands. But I do not have any more to say about this. It is worth seeing whether the theory is attractive in the rough, before refining the detail.

Median utilitarianism avoided the problem of utility monsters, and so does mode utilitarianism. Increasing the happiness of someone above the mode is only of significance if it impacts in some way upon the mode. Otherwise, it is an irrelevance. Here, then, the position mirrors that of median utilitarianism in which the attempt to reformulate the utility monster objection turns it into the problem

of victimisation. We must now discuss this, and here the two versions of mode utilitarianism need separate treatment.

Let us take, first, irreflexive mode utilitarianism (in its hybrid form) in which we band people, initially, by social group. Thus we are told to make the largest group - - we have chosen the C2s - - as happy as possible. This obviously leads to severe problems of victimisation. The rest of the population - - the As, Bs, C1s, Ds and Es - - can be made to suffer any ill provided it raises the position of the C2s by the slightest amount. This is worse than the problem of victimisation as it afflicts mean utilitarianism, and arguably worse than the way the problem afflicts median utilitarianism. For we could only raise the median by raising everything currently above the median so this ensures that at least the majority must benefit from sacrifice. But as the mode need not be the majority, this understanding of mode utilitarianism allows, even requires, the sacrifice of the majority for the majority.

As an aside, we might wonder whether irreflexive mode utilitarianism is the morality of certain revolutionary groups. A certain social class - - probably the largest - - is chosen, and then everything is done, without restraint, the further the interests of that class. This illustration, I think, demonstrates the unattractiveness of the theory.

How about reflexive mode utilitarianism? We saw that the best way of thinking about this is by means of a bar graph, in which bands of happiness are plotted on the x axis, ranked from lower to higher, and numbers of people are plotted on the y axis. The mode band will be the highest bar. To maximise the mode is to find the distribution such that it has its highest column furthest to the right. Bearing in mind that we are not considering population change cases the total people, and thus the total number of people, should be the same in each graph (representing different possible worlds). In principle there is no reason why we shouldn't be faced with a situation in which sacrificing some will increase the happiness of the mode. This can be represented in graphical form very easily. But what would be an example? The standard scapegoating examples used against classical (and hence mean) utilitarianism will do. We might move the mode up a notch from anxious to contented by hanging a few innocents, and thus making a lot of people sleep more easily. So some sacrifices are clearly mandated by reflexive mode utilitarianism. But how serious is this? Bearing in mind that changes to happiness level changes potentially changes the banding, and thus the mode too, the situation is complex.

Take the case where the mode is currently contented. We realise that we can raise these people to the level of cheerful by undertaking policies that reduce the entire remainder of the population to the level of anxiety. But by doing this it is very likely that we have lowered the mode, not raised it. For the anxious group may well be larger than the cheerful group, and thus would be the new mode. Clearly, then, there are going to be limits to the sacrifices demanded.

In reply, it will be said that the ideal policy is one that sacrifices others to the mode group, but not in such a way that a new, lower, mode forms. Here, though, we might have a fairly precarious balancing act to do. For a slightly misjudged social policy may wrongly create a new low mode. Given the uncertainties of planning, we would want to leave quite a margin for error. This might give some comfort. Yet anti-consequentialist are rarely comforted by such contingencies of the pragmatics of application.

Victimisation is clearly an area in which there is more to say, and, indeed, we may wish to formulate more versions of mode-utilitarianism, so see if we can find a more resistant strain, but I will not do so here, as the general message is clear. Reflexive mode utilitarianism will permit, even require, some sacrifices in some cases. How serious this is, and how it compares to other forms of average utilitarianism remains to be seen.

So far, much of the discussion has been technical in nature: how can we define alternative forms of average utilitarianism, and how well do they fare against standard objections? However, there remains the obvious question of why we should be interested in any of this. Pure consequentialism is not in vogue, and so explorations of its possible forms may seem a purely theoretical exercise. Yet we are seeing some interest in consequentialism as part of a broader theory, both in moral and political philosophy. For example, it is common now in discussions of equality to make some concessions towards efficiency, and to utility. Yet the possibilities of combining anything at all with mode or median utilitarianism have not been explored as those theories have not been put on the table. It could be that there is a rich field of enquiry here. Or not.