Inductive Skepticism and the Probability Calculus I: Popper and Jeffreys on Induction and the Probability of Law-Like Universal Generalizations*

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1. Introduction. Attempts to utilize the probability calculus to prove or disprove various inductive or inductive skeptical theses are, I believe, highly problematic. Inductivism and inductive skepticism are substantive (logically consistent) philosophical positions that do not allow of merely formal proofs or disproofs. Often the problems with particular alleged formal proofs of inductive or inductive sceptical theses turn on subtle technical considerations. In the following I highlight such considerations in pointing out the flaws of two proofs, one an alleged proof of an inductive sceptical conclusion due to Karl Popper, the other an alleged proof of an inductivist thesis originally due to Harold Jeffreys and later advocated by John Earman. Surprisingly, in examining Popper's argument it is shown that certain apparently weak premises, often embraced by both inductivists and deductivists, lend themselves to inductive conclusions. However it is argued that those premises are still philosophically substantive and not amenable to a purely formal demonstration. The lesson to be learnt here is twofold. First, we need to be very careful in determining which formal theses entail, and which are entailed by, inductive skepticism and inductivism. Second, we need

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to take great care in laying out and examining the assumptions presumed in formal arguments directed for and against such formal theses. In a follow up article I will consider various attempts by David Stove and Karl Popper and David Miller to identify the exact content of inductive skepticism and propose a new identification based on the theory of content developed in Gemes 1994. Finally I will compare this new version with that proposed recently in Alberto Mura 1990.

2. Popper's Proof of the 0 Probability of Law Statements. Notoriously, Popper argues that every (non-tautologous) law-like universal generalization (hereafter LUG) has a prior probability of 0.[1] From this claim it follows that there can be no inductive confirmation, or, at least, probabilistic support, for LUGs, for it follows from the Probability Calculus that where a statement h has a prior probability of 0, for any evidence e, the posterior probability of h on e is 0.[2] Popper produces a strictly a priori argument for that claim that LUGs have 0 prior probability. Here are the nuts and bolts of Popper's formal argument.

Let L be any LUG and let $E_1, E_2, \ldots,$ be distinct instances of L.[3] Now since for any $i \varepsilon N$, $L \vdash E_i$, $P(L) \leq \lim_{n \to \infty} P(E_1 \ \& \ E_2 \ \& \ldots \& E_n)$.

Assume **The Principle of Independence of Instances of LUGs** (hereafter, Instantial Independence), that is, that distinct instances of a LUG are probabilistically independent, that is, for any n, $P(E_1 \ \& \ E_2 \ \& \ldots \& \ E_n) = P(E_1) \times P(E_2) \times \ldots \times P(E_n)$. Then, further assuming **The Principle of Regularity for Instances of LUGs** (hereafter, Instantial Regularity), that is, that the instances of a LUG have a prior probability of less than 1, in particular, for any $i \varepsilon N$, $P(E_i) < 1$, and **The Principle of the Simple Exchangeability of Instances of LUGs** (hereafter, Simple Exchangeability of Instances), that is, for any $i, j \varepsilon N$, $P(E_i) = P(E_j)$, it follows that

1. A classic statement of Popper's argument occurs in Appendix *vii of Popper 1972. In fact, while in the following we talk of law-like universal generalizations, this is in deference to Popper's talk of law statements. No substantive position about the nature of laws need be here assumed. Talk of law-like universal generalizations could be replaced by talk of non-tautologous, non-contradictory universal statements without any effect on subsequent arguments.

2. Where the probability of e is itself 0 the posterior probability of h on e is 0 or undefined.

3. E_1, E_2, etc., are instances in the logician's sense. That is, where L is a universal generalization (without vacuous quantifiers), an instance results from eliminating the outer most quantifier of L and uniformly replacing occurrences of the variable bound by that quantifier with an individual constant.

$$\lim_{n\to\infty} P(E_1 \&. .\& E_n) = \lim_{n\to\infty} P(E_1)^n = 0.$$ [4,5]

Therefore $P(L) \leq 0$. And since for any statement S, $P(S) \geqslant 0$, $P(L) = 0$. Let us call this the "0 Probability of LUGs Argument."

3. A Problem for the Principle of Instantial Irrelevance. Typically inductivists have attacked Popper's assumption that distinct instances are probabilistically independent. In particular, they claim that Instantial Independence begs the question against inductivism. In fact, it is argued in Gemes (1989) that Instantial Independence itself conflicts with other tenets of Popperian philosophy. In other words, one does not have to be an inductivist to reject Instantial Independence. Here is a generalized version of that argument. Instantial Independence entails

$$P(\alpha a_1/\alpha a_2) = P(\alpha a_1), \qquad (1)$$

where $\ulcorner \alpha a_1 \urcorner$ and $\ulcorner \alpha a_2 \urcorner$ are distinct instances of the LUG $\ulcorner (x)(\alpha x) \urcorner$. Note, for our purpose α may well be a conditional predicate of the form 'if x has property P then x has property Q'. Now clearly any Popperian inductive sceptic will endorse the claim

$$P(\alpha a_1/(a_1 \neq a_2) \& \alpha a_2) = P(\alpha a_1). \qquad (2)$$

By substituting '$((a_1 = a_2)\&\alpha a_1)V((a_1 \neq a_2)\&\alpha a_1))$' for the first occurrence of its logical equivalent 'αa_1' in (1) we get,

$$P(((a_1 = a_2)\&\alpha a_1)V((a_1 \neq a_2)\&\alpha a_1)/\alpha a_2) = P(\alpha a_1). \qquad (3)$$

By the standard disjunction principle for mutually exclusive dis-

4. Note, Simple Exchangeability for Instances of LUGs is not equivalent to (full) Exchangeability, as introduced by De Finetti. For more on this see Section 6, below.

5. Popper does not in fact explicitly make the assumption of Simple Exchangeability of Instances of LUGs. Strictly speaking, neither this assumption nor even Regularity is needed for the proof. What is needed is some set of assumptions which entails

(A) For any LUG L, where $\{E_1, E_2, \ldots\}$ is the set of the infinite distinct instances of L, there is some subset S of infinite cardinality of $\{E_1, E_2, \ldots\}$ and some number r such that $r < 1$ and for any E_i, if $E_i \varepsilon S$ then, $P(E_i) \leq r$.

In fact, Popper's presentation of the proof in his 1972 makes no assumption ruling out the possibility that as n increases $P(E_n)$ approaches 1. Simply replacing Simple Exchangeability for Instances of LUGs with the claim that as n increases $P(E_n)$ approaches 1 would render the argument for the 0 probability of Laws invalid. However that claim is so far-fetched it would be fairly uncharitable to fault Popper for ignoring it. Note, replacing the assumptions of Simple Exchangeability of Instances of LUGs and Regularity with (A) itself would complicate the details but not the end results of the various arguments below.

juncts—where p and q are mutually exclusive, P(pvq/r) = P(p/r) + P(q/r)—we get from (3),

$$P((a_1 = a_2)\&\alpha a_1/\alpha a_2) + P((a_1 \neq a_2)\&\alpha a_1/\alpha a_2) = P(\alpha a_1). \quad (4)$$

By the standard conjunction principle—P(p&q/r) = P(p/r) × P(q/p&r)—we get from (4),

$$P(a_1 = a_2/\alpha a_2) \times P(\alpha a_1/(a_1 = a_2)\&\alpha a_2) + P(a_1 \neq a_2/\alpha a_2) \times P(\alpha a_1/(a_1 \neq a_2)\&\alpha a_2) = P(\alpha a_1). \quad (5)$$

Now since $(a_1 = a_2)\&\alpha a_2 \vdash \alpha a_1$, $P(\alpha a_1/(a_1 = a_2)\&\alpha a_2) = 1$. So from (5) we get,

$$P(a_1 = a_2/\alpha a_2) \times 1 + P(a_1 \neq a_2/\alpha a_2) \times \mathbf{P(\alpha a_1/(a_1 \neq a_2)\&\alpha a_2)} = P(\alpha a_1). \quad (6)$$

Substituting by (2) for the bold-faced expression in (6), we get

$$P(a_1 = a_2/\alpha a_2) + P(a_1 \neq a_2)/\alpha a_2) \times P(\alpha a_1) = P(\alpha a_1). \quad (7)$$

Dividing throughout (7) by $P(\alpha a_1)$ we get

$$P(a_1 = a_2/\alpha a_2)/P(\alpha a_1) + P(a_1 \neq a_2)/\alpha a_2) = 1. \quad (8)$$

Now as an instance of the standard negation principle—P(p/q) + P(~p/q) = 1—we have

$$P(a_1 = a_2/\alpha a_2) + P(a_1 \neq a_2/\alpha a_2) = 1. \quad (9)$$

From (8) and (9) it follows that

$$P(a_1 = a_2/\alpha a_2)/P(\alpha a_1) = P(a_1 = a_2/\alpha a_2). \quad (10)$$

From (10), dividing through by $P(a_1 = a_2/\alpha a_2)$, assuming $P(a_1 = a_2/\alpha a_2) \neq 0$, we get

$$P(\alpha a_1) = 1. \quad (11)$$

Let us call the above argument, for reasons soon to become apparent, The Popperian Argument against Instantial Regularity, Identity, or Instantial Independence, for short, PARII. Now the conclusion of PARII, namely $P(\alpha a_1) = 1$, is not consistent with The Principle of Instantial Regularity used in the 0 Probability of LUGs Argument above. To avoid that conclusion one must deny one of the premises or assumptions of the argument. Clearly the option of denying the inductive sceptical thesis (2), $P(\alpha a_1/(a_1 \neq a_2)\&\alpha a_2) = P(\alpha a_1)$, is not open to Popperians. On the other hand to deny (1), $P(\alpha a_1/\alpha a_2) = P(\alpha a_1)$, is to deny the Principle of Instantial Independence which is crucial to the 0 Probability of LUGS Argument. To avoid this problem a Popperian might deny the assumption $P(a_1 = a_2/\alpha a_2) \neq 0$ used in PARII. Without the claim that $P(a_1 = a_2/\alpha a_2) \neq 0$ the move from (10) to (11) would be invalid. Indeed Miller (1990), replying to Gemes (1989), claimed that Popperians are committed to the claim that $P(a_1 = a_2) = 0$ and hence, a fortiori, to its consequence $P(a_1 = a_2/\alpha a_2) = 0$.[6] The claim that $P(a_1 = a_2) = 0$ need not be a gratuitous piece of dogmatism but rather simply reflect a somewhat parochial insistence that the language we are working in uses uniquely designating individual constants. The problem with this reply is that it does not sit well with The Principle of Instantial Regularity of the 0 Probability of LUGs Argument. Let us consider why.

4. A Popperian Argument for A Priori Infinite Populations. For the 0 Probability of LUGS Argument to work there must be an infinite number of distinct instances E_1, E_2, etc., for any arbitrary LUG L. So there must be an infinite number of individual constants in the language in which L and its instances are expressed. Now typically a LUG will be (logically equivalent to) a universal conditional of the form $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$, where both σ and ϕ are non-contradictory, non-tautologous predicates. Now suppose we allow that there may only be a finite numbers of σs. Then, given the Popperian assumptions that there are an infinite number of uniquely naming constants c_1, c_2, etc, that instances are probabilistically independent and exchangeable, it follows that the prior probability that any arbitrary constant c_i, $i\varepsilon N$, names a σ will be 0. In other words, given that there may only be a finite number of σs, from the Popperian assumptions mentioned above, it follows that $P(\sigma c_i) = 0$ and hence $P(\sim\sigma c_i) = 1$. Further, under those assumptions, and since $\sim\sigma c_i \vdash \sigma c_i \rightarrow \phi c_i$, it follows that $P(\sigma c_i \rightarrow \phi c_i) = 1$. But this conclusion contradicts The Principle of Instantial Regularity of Popper's 0 Probability of LUGs Argument. The assumption, that all instances of a generic LUG $\ulcorner(x)(\sigma \rightarrow \phi x)\urcorner$ have a prior probability less than 1 is only acceptable, given Popper's other assumptions, if we a priori accept that there are infinitely many σs. Note, it will not do simply to assume that our domain of quantification contains an infinite number of individuals be they in Popper's words "infinite with respect to the number of distinguishable things, or of spatio-temporal regions" (Popper 1972, 363). What Popper needs for his argument to go through for all LUGs of the form $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$ is the assumption that there is an infinite number of σs. Where the domain of quantification is infinite but we allow there may only be a finite number of σ's in that

6. A slight and irrelevant change of notation from that of Gemes 1989 and Miller 1990 is used here.

domain then, where our language contains an infinite number of uniquely designating exchangeable individual constants and instantial independence holds, the probability that arbitrary individual constant c_i names a σ will be 0.

It is not being claimed here that the consequence of The Principle of Instantial Regularity that for any LUG ⌈(x)(σx → φx)⌉ and any individual constant c_j, $P(\sigma c_j \rightarrow \phi c_j) < 1$ fails if we assume a priori that there are only finitely many σs and that instances are exchangeable and independent. This latter claim is of no great interest since the a priori claim that there are finite σs is inadmissible. Rather the claim is that the conjunction of the claims

Uniqueness of Reference: $(a_j)(a_k)(j \neq k \rightarrow P(a_j = a_k) = 0)$, (12)

Instantial Independence of Antecedents of Conditional LUGS: For any conditional LUG of the form '(x)(σx → φx)', (13) where $a_1, \ldots, a_m$ are distinct individual constants, $P(\sigma a_1 \,\&\, .. \,\&\, \sigma a_m) = P(\sigma a_1) \times .. \times P(\sigma a_m)$,

Instantial Regularity for Instances of the Antecedents of Conditional LUGs: Where 'σ_j' is an instance of the antecedent (14) of a conditional LUG of the form '(x)(σx → φx)', $P(\sigma a_j) > 0$,

Simple Exchangeability of Antecedents of Conditional LUGs: Where c_j and c_k are individual constants, for any conditional (15) LUG of the form '(x)(σx → φx)', $P(\sigma c_j) = P(\sigma c_k)$,

entail, given the standard axioms of probability,

The probability that there are an infinite number of σs is 1.[7] (16)

To prove this it will suffice the prove that the conjunction of (12) to (15) above entails

For any (finite) m the probability there are at least m σs is 1, (16*)

since (16*) clearly entails (16.)[8]

7. Strictly speaking, one does not need (Partial) Instantial Regularity or Simple Exchangeability of Antecedents of LUGs to get this conclusion. (14) and (15) could be replaced, *salva validitate,* with the premise

Where $\{\sigma a_1, \sigma a_2, \ldots\}$ is the set of all the distinct instances of (x)(σx) there is some subset S of infinite cardinality of $\{\sigma a_1, \sigma a_2, \ldots\}$ and some number n such that $n > 0$ and for any E, if $E \varepsilon S$ then, $P(E) \geq n$.

8. Consider the infinite set of statements {'There is at least 1 dog', 'There are at least 2 dogs', ... , 'There are at least n dogs', ...}. Where each member of this set is true the statement 'There are an infinite number of dogs' is also true. So where each member of this set has a probability of 1, and hence the whole set has a probability of 1, the

Before providing the proof, it is worth noting that (13) is a consequence of the Popperian Principle of Instantial Independence given that '(x)σx' itself counts as a LUG. To deny that '(x)σx' counts as a LUG is to put far too much weight on the notion of what counts as a LUG (Cf. endnote 1). In any case, a Popperian cannot deny (13) without endorsing the thesis

For some value of n, $n \leq m$, $P(\sigma a_{n+1}/\sigma a_1\& \ldots \&\sigma_n) \neq P(\sigma a_{n+1})$. Yet where the Popperian (12) is true this thesis is frankly inductivist and hence not open to Popperians. (14) is a consequence of the Popperian Principle of Instantial Regularity. The later entails that for generic conditional LUGs of the form '(x)(σx → ϕx)', $P(\sigma a_j \rightarrow \phi a_j) < 1$. This, in turn, entails $P(\sim\sigma a_j) < 1$, and hence, $P(\sigma a_j) > 0$. Finally Simple Exchangeability of Instances entails Simple Exchangeability of Antecedents of Conditional LUGs given that '(x)(σx)' counts as a LUG. In any case, to accept Simple Exchangeability of Instances while denying Simple Exchangeability of Antecedents of Conditional LUGs is simply too bizarre to warrant serious consideration.

Proof That (12)–(15) Entail (16)*

Assume (12)–(15)

Let m be any arbitrary (finite) number.

Now in a language where by stipulation names name uniquely, that is where (12) holds, any m lengthed conjunction of distinct atomic wffs of the form ⌜σc⌝, where σ is a predicate and c is an individual constant, entails the claim that there are at least m σs. For instance '$\sigma a_1\&\sigma a_2$' entails 'There exist at least 2 σ's'. Similarly, any n-lengthed disjunction of such m lengthed conjunctions entails that there are at least m σs. For instance, '$((\sigma a_1\&\sigma a_2)v(\sigma a_3\&\sigma a_4))$' entails 'There are at least 2 σs'. So the probability of any n-lengthed disjunction of such m-lengthed conjunctions must be less than or equal to the probability of the claim 'There are at least m σs'.[9]

We now show that

$$\lim_{n\rightarrow\infty} P((\sigma a_1 \& \ldots \& \sigma a_m)v \ldots v(\sigma a_{(n-1)m+1} \& \ldots \&\sigma a_{nm}) = 1.$$

For notational convenience let 'σ_m' abbreviate the claim '$(\sigma a_1 \& \ldots \& \sigma a_m)$', '$\sigma_{2m}$' abbreviate the claim '$(\sigma_{am+1}\& \ldots \& \sigma a_{2m})$', etc. Let $P(\sigma_m) =$

statement 'There are an infinite number of dogs' has a probability 1.

9. The crucial point here is that where $P(\sigma a_1\&\sigma a_2) = r$ and, $P(a_1 = a_2) = 0$, hence $P(a_1 \neq a_2) = 1$, $P(\exists x)(\exists y)(\sigma x\&\sigma y\&(x \neq y)) \geq r$, since $(\sigma a_1\&\sigma a_2)\ \&\ (a_1 \neq a_2) \vdash (\exists x)(\exists y)(\sigma x\&\sigma y\&(x \neq y))$. By the same token, where $P((\sigma a_1 \& \ldots \& \sigma a_m)\, v \ldots v(\sigma a_{(n-1)m+1} \& \ldots \&\sigma a_{nm}) = s$ and $(a_j)(a_k)(j \neq k \rightarrow P(a_j = a_k) = 0)$ it follows that P(There are at least m σs) ≥ s.

b. Then, given (13) and (15), it follows that $P(\sigma_m) = (\sigma_{2m}) = \ldots = (\sigma_{nm}) = b$.

$$
\begin{aligned}
\text{Now, } P(\sigma_m \vee \sigma_{2m}) &= P(\sigma_m) + P(\sigma_{2m}) - P(\sigma_m \;\&\; \sigma\alpha_{2m}) \\
&= b + b - P(\sigma_m) \times P(\sigma_{2m}/\sigma_m) \\
&= b + b - P(\sigma_m) \times P(\sigma_{2m}) \qquad \text{[by (13)]} \\
&= 2b - (b \times b) \\
&= 2b - b^2 \\
&= 1 - (1 - b)^2.
\end{aligned}
$$

More generally for any n, where $n > 0$,

$$P(\sigma_m \vee \ldots \vee \sigma_{nm}) = 1 - (1 - b)^n.$$

This we show by induction.

Basis Step:

$$P(\sigma_m) = b$$
$$1 - (1 - b)^1 = b$$

Therefore where $n = 1$,

$$P(\sigma_m \vee \ldots \vee \sigma_{nm}) = 1 - (1 - b)^n.$$

Inductive Step:

$$\text{Assume: } P(\sigma_m \vee \ldots \vee \sigma_{nm}) = 1 - (1 - b)^n.$$
$$\text{To Prove: } P(\sigma_m \vee \ldots \vee \sigma_{(n+1)m}) = 1 - (1 - b)^{n+1}.$$

$$
\begin{aligned}
P(\sigma_m \vee \ldots \vee \sigma_{(n+1)m}) &= P(\sigma_m \vee \ldots \vee \sigma_{nm} \vee \sigma_{(n+1)m}) \\
&= P(\sigma_m \vee \ldots \vee \sigma_{nm}) + P(\sigma_{(n+1)m}) - P((\sigma_m \vee \ldots \vee \sigma_{nm}) \& (\sigma_{(n+1)m})) \\
&= 1 - (1 - b)^n + b - [P(\sigma_m \vee \ldots \vee \sigma_{nm}) \times P(\sigma_{(n+1)m}/\sigma_m \vee \ldots \vee \sigma_{nm})] \\
&= 1 - (1 - b)^n + b - [(1 - (1 - b)^n) \times P(\sigma_{(n+1)m})] \text{[by (13)]}^{10} \\
&= 1 - (1 - b)^n + b - [(1 - (1 - b)^n) \times b] \\
&= 1 - (1 - b)^n + b - [b - b(1 - b)^n] \\
&= 1 - (1 - b)^n + b - b + b(1 - b)^n \\
&= 1 - (1 - b)^n + b(1 - b)^n \\
&= 1 + b(1 - b)^n - (1 - b)^n \\
&= 1 - (1 - b)(1 - b)^n \\
&= 1 - (1 - b)^{n+1}
\end{aligned}
$$

10. The proof of claim that (13) entails that $P(\sigma_{(n+1)m}/\sigma_m \vee \ldots \vee \sigma_{nm}) = P(\sigma_{(n+1)m})$ I leave as an exercise to the reader. Hint: Consider the claim $P(\sigma a_3/\sigma a_2 \vee \sigma a_1) = P(\sigma a_3)$. Now

Therefore $\lim_{n\to\infty} P(\sigma_m \text{ v} \,..\, \text{v } \sigma_{nm}) = \lim_{n\to\infty} 1 - (1 - b)^n$.

Now from (13), (14) and (15) if follows that $0 < b \leq 1$,[11] so

$$\lim_{n\to\infty} 1 - (1 - b)^n = 1.$$

Therefore $\lim_{n\to\infty} P(\sigma_m \text{ v} \,...\, \text{v } \sigma_{nm}) = 1$

5. Popper's Dilemma. To make the argument for the claim that the prior probability of any LUG is 0 Popper needs to assume Instantial Independence, Instantial Regularity and Simple Exchangeability of Instances. Instantial Independence prima facie entails the Instantial Independence of Antecedents of Conditional LUGS, that is, (13) above. At any rate we saw above that Popperians are in no position to deny (13) without lapsing into inductivism. Instantial Regularity entails Instantial Regularity for Instances of the Antecedents of Conditional LUGs, that is, (14) above. Simple Exchangeability of Instances entails Simple Exchangeability of Instances of Antecedents, that is, (15) above, or at least is unacceptable without it. The PARII argument above demonstrates that a commitment to Instantial Independence can only avoid inductivism at the price of rejecting Instantial Regularity or accepting the Uniqueness of Reference claim, (12) above. Popper notoriously rejects all forms of inductivism. So, given his acceptance of Instantial Regularity, his only remaining option is that, favored by Miller, of accepting (12). But, as demonstrated above, this combination of claims, (12), (13), (14) and (15) saddles him with an inadmissible a priori commitment to the claim that there are infinitely many σs, where σ is any predicate capable of serving as the antecedent term of a simple conditional LUG.

In other words, Popper's 0 Probability of LUGs Argument thus faces the following dilemma: (i) if we allow that in the relevant language individual constants may name non-uniquely, the combination of Instantial Independence and Instantial Regularity is tenable only at the

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$P(\sigma a_3/\sigma a_2 \text{ v } \sigma a_1) = [P(\sigma a_3) \times P(\sigma a_2 \text{ v } \sigma a_1/\sigma a_3)] \div P(\sigma a_2 \text{ v } \sigma a_1) = [P(\sigma a_3) \times (P(\sigma a_2/\sigma a_3) + P(\sigma a_1/\sigma a_3) - P(\sigma a_2 \& \sigma a_1/\sigma a_3))] \div P(\sigma a_2 \text{ v } \sigma a_1)$. Now given (13), $P(\sigma a_2/\sigma a_3) = P(\sigma a_2)$, $P(\sigma a_1/\sigma a_3) = P(\sigma a_1)$ and $P(\sigma a_2 \& \sigma a_1/\sigma a_3) = P(\sigma a_2 \& \sigma a_1)$. So, given (13), $P(\sigma a_3/\sigma a_2 \text{ v } \sigma a_1) = [P(\sigma a_3) \times (P(\sigma a_2) + P(\sigma a_1) - P(\sigma a_2 \& \sigma a_1))] \div P(\sigma a_2 \text{ v } \sigma a_1) = [P(\sigma a_3) \times (P(\sigma a_2 \text{ v } \sigma a_1))] \div P(\sigma a_2 \text{ v } \sigma a_1) = P(\sigma a_3)$.

11. By stipulation, $b = P(\sigma a_1 \,\&\, ..\, \&\, \sigma a_m)$. From the axioms of probability it follows that $P(\sigma a_1 \,\&\, ..\, \&\, \sigma a_m) \leq 1$. So we merely need prove that $P(\sigma a_1 \,\&\, ..\, \&\, \sigma a_m) > 0$. Now (13) entails that $P(\sigma a_1 \,\&\, ..\, \&\, \sigma a_m) = P(\sigma a_1) \text{ x } .. \text{ x } P(\sigma a_m)$. (15) entails that $P(\sigma a_1) = P(\sigma a_2) = \,..\, P(\sigma a_m)$. So $P(\sigma a_1 \text{ x } .. \text{ x } \sigma a_m) = P(\sigma a_1)^m$. Now (14) entails that $P(\sigma a_1) > 0$, hence, since by stipulation m is a finite number, $P(\sigma a_1)^m > 0$.

price of inductivism;[12] (ii) if we insist that in the relevant language the individual constants name uniquely, the consequence of Instantial Regularity that every instance of the generic LUG $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$ has a probability less than unity combined with Simple Exchangeability of Instances and Instantial Independence is a priori acceptable only if we assign a priori a probability of 1 to the claim that there are infinitely many σs.

The main upshot of this is that, even granting all Popper's claims about independence of individuals and instances, we are not compelled to accept that LUGs such as 'All ravens are black' and 'All electrons repel each other' have an a prior probability of 0 save we accept a priori that there are an infinite number of ravens and electrons. Such assumptions are presumably indefensible a priori, as they are a posteriori, to both inductivists and deductivists alike.

Deductivists who eschew such a priori assumptions about population sizes may react here by simply rejecting the Popperian claim that LUGs have a prior probability of 0. Popperians, on the other hand, cannot take such a simple route. At least, in as much as they, following Popper, are committed to Simple Exchangeability of Instances, Instantial Regularity, Instantial Independence, and the Uniqueness of Reference of individual constants, they are committed to the implausible claim that for any predicate σ capable of serving as the antecedent of a LUG $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$ there is probability of 1 that there are infinitely many σs.

6. Simple Exchangeability and Induction. Perhaps some enthusiastic readers will think that there is a proof of induction in all this. Thus they may reason: Given the total unacceptability of the infinite populations thesis and the trivial thesis of the uniqueness of reference (a matter of selecting an appropriate language) all we need to get induction (where names name uniquely Instantial Dependence is a form of inductivism; for more on this see Section 8 below) is the presumably uncontroversial principles of Instantial Regularity and Simple Exchangeability of Instances. The reasoning here is that since we have seen that (12)–(15) entail the unacceptable (16) and (12), (14) and (15) are beyond reproach the only viable option is to reject (13), and hence to reject Instantial Independence. If we were to raise questions about

12. More particularly, as demonstrated by PARII, the combination of $P(\alpha a_1 = \alpha a_2/\alpha a_2) \neq 0$, $P(\alpha a_1/\alpha a_2) = P(\alpha a_1)$, and $P(\alpha a_1) < 1$ is inconsistent with the inductive sceptical thesis $P(\alpha a_1/(a_1 \neq a_2) \,\&\, \alpha a_2) = P(\alpha a_1)$. Note, this only temporarily leaves open the possibility that the Popperian might not actually endorse $P(a_1 = a_2) = 0$ but only claim that $P(a_1 = a_2/\alpha a_2) = 0$. For, by a similar argument, basically substituting '$\sim\alpha a_1$' for 'αa_1' and '$\sim\alpha a_2$' for 'αa_2' throughout PARII, the Popperian will be forced to endorse $P(a_1 = a_2/\sim\alpha a_2) = 0$. This together with $P(a_1 = a_2/\alpha a_2) = 0$ entails $P(a_1 = a_2) = 0$.

(15), that is, Simple Exchangeability of Antecedents of Instances, they may take courage from the fact that this type of Exchangeability is prima facie far less problematic than De Finetti's Full Exchangeability.[13] Very true. For instance, Full Exchangeability, that is, the claim that for any statements S and S′ where S′ is solely the result of a permutation of some of the individual constants of S, (P(S) = P(S′), commits one to the claim

$$P(\sigma a_{11}/\sigma a_1\&\sim\sigma a_2\&\sigma a_3\&\sim\sigma a_4\&\sigma a_5\&\sim\sigma a_6\&\sigma a_7\&\sim\sigma a_8 \&\sigma a_9\&\sim\sigma a_{10}) = P(\sigma a_{12}/\sigma a_1\&\sim\sigma a_2\&\sigma a_3\&\sim\sigma a_4\&\sigma a_5 \&\sim\sigma a_6\&\sigma a_7\&\sim\sigma a_8\&\sigma a_9\&\sim\sigma a_{10}). \qquad (17)$$

Now one might reject (17) on the grounds that one takes the evidence statement '$(\sigma a_1\&\sim\sigma a_2\&\sigma a_3\&\sim\sigma a_4\&\sigma a_5\&\sim\sigma a_6\&\sigma a_7\&\sim\sigma a_8\&\sigma a_9\& \sim\sigma a_{10})$' as giving some grounds for believing that all individual constants with odd numbered subscripts name σs and all individual constants with even numbered subscripts name non σs. In contrast, the Simple Exchangeability of Antecedents of Instances does not commit one to (17). Still, even granting that Full Exchangeability is more controversial than Simple Exchangeability of Antecedents of Instances, I strongly urge the reader to reconsider the claim that Simple Exchangeability of Antecedents of Instances is itself totally uncontroversial. Or, rather, I urge the reader to carefully consider to what extent Simple Exchangeability of Antecedents of Instances is less controversial than induction itself.

To pinpoint what is, perhaps, questionable about the Simple Exchangeability of Antecedents of Instances it will help to focus on the following principle;

> The Principle of Simple Exchangeability: For any constants a_j and a_k, $P(\alpha a_j) = p(\alpha a_k)$.

Clearly this Simple Exchangeability is not something we should endorse for any predicate whatsoever. For instance, suppose the language L includes the primitive predicates 'M', and 'S' where 'M' and 'S' respectively have the sense of the English predicates 'is metal' and 'is silver'. Further suppose L contains the defined predicate 'H' introduced as follows;

$$Hx =_{df.} (x = a_1\&Mx\&Sx) \vee (x \neq a_1\&Mx).$$

13. De Finetti did not actually endorse Full Exchangeability. Rather he demonstrated some of the fascinating consequences of acceptance of this and other notions of Exchangeability. Cf., for instance, "Foresight: Its Logical Laws, Its Subjective Sources", in Kyburg Jr. and Smokler 1964.

In this case accepting Uniqueness of Reference and Simple Exchangeability applied to 'H' would commit us to the prima facie unacceptable claim $P(Sa_1/Ma_1) = 1$.

Rather than adopting Simple Exchangeability simpliciter we should at most accept Simple Exchangeability on a predicate by predicate basis. As a matter of fact, I think we do generally take the basic predicates of our preferred languages to obey Simple Exchangeability. To a certain extent, obeying Simple Exchangeability is indeed a sine qua non for being a bona fide basic predicate.[14] From the above we know that that is tantamount to saying that being a predicate suitable for induction is a sine qua non for being a bona fide basic predicate. However the claim that genuine basic predicates obey Simple Exchangeability is one that needs substantive philosophical justification and is not susceptible of a merely formal proof.

It has long been known that (full) Exchangeability combined with Regularity rules out Instantial Independence.[15] However that kind of Exchangeability is not without its critics. Above we have seen that the much weaker notion of Simple Exchangeability of Antecedents of Instances does not sit well with Instantial Independence for a language whose individual constants name uniquely. That, I believe, is as close to a merely formal proof of induction that we will get.

7. The Earman-Jeffreys Proof that the Positive Probability of LUGs entails Inductivism. It is important to note the limits of the argument of Section 3–5 above. It is not an argument for the conclusion that all or some LUGs must have a prior probability greater than 0. Rather, I claim to show that Popper's a priori argument for the claim that all LUGs have a probability of 0 is flawed. How does this relate to the question of inductive skepticism? In fact both a hardcore inductivist and hardcore inductive sceptic are free to accept or reject the claim that LUGs have 0 probability. For instance an inductivist while assigning a 0 probability to the claim '$(x)(Rx \rightarrow Bx)$'—say, the claim that all ravens are black—might still claim that '$Ra_1 \& Ba_1 \& a_1 \neq a_2$' is favorably relevant to '$Ra_2 \rightarrow Ba_2$'; this is indeed the position taken in Carnap 1962. On the other hand, an inductive sceptic might assign a probability greater than 0 to '$(x)(Rx \rightarrow Bx)$' while claiming, among other things, that, for instance, '$Ra_1 \& Ba_1 \& a_1 \neq a_2$' is irrelevant to '$Ra_2 \rightarrow Ba_2$'.

14. One might further claim that obeying Simple Exchangeability is a sine qua non for being a predicate capable of figuring in genuine law statements.

15. See, for instance, Jurgen Humburg's "The Principle of Instantial Relevance" and Haim Gaifman's "Applications of De Finetti's Theorem to Inductive Logic". Both appear in Carnap and Jeffrey 1971.

John Earman (1985), following Harold Jeffreys (1957), has advanced an interesting argument for the conclusion that, contra the claim above, the rejection of the claim that LUGs have 0 probability entails a form of inductivism.[16] Here is a version of the Earman-Jeffreys argument.

Consider the universal generalization '(x)(αx)' and its instances, 'αa_1', 'αa_2' etc. Note, for our purpose α may well be a conditional predicate of the form 'if x has property P then x has property Q'. Now for any $n \varepsilon N$,

$$P((x)(\alpha x)/(\alpha a_1 \mathbin{\&} .. \mathbin{\&} \alpha a_{n+1})) = \frac{P((x)(\alpha x))}{P(\alpha a_1) \text{ x } .. \text{ x } P(\alpha a_{n+1}/(\alpha a_1 \mathbin{\&} .. \mathbin{\&} \alpha a_n))}. \quad (18)$$

Now if $P((x)(\alpha x)) > 0$ then as n increases the denominator of the right hand side of (18) will eventually become smaller than the numerator yielding, contra the axioms of probability, a total value greater than 1, unless

$$\lim_{n \to \infty} P(\alpha a_{n+1}/(\alpha a_1 \mathbin{\&} .. \mathbin{\&} \alpha a_n)) = 1. \quad (19)$$

Where (19) is true Earman says '(x)(αx)' is "weakly projectible in the future moving instance sense" (1985, 522). For Earman (19) is a type of instance induction.[17]

Yet does acceptance of (19) really amount to some form of inductivism? I will soon argue that a deductivist can accept (19) without in any way compromising his deductivist scruples.

8. A Deductivist Argument For Earman's Weak Projectibility. Before commencing with our central argument, it is perhaps worth noting that a deductivist might accept (19) simply because he rejects the Regularity Principle $(n)P(\alpha a_n) < 1$. For instance, he might accept the claim, where

16. We take the liberty of assuming that LUGS form a subclass of the set of universal hypotheses which are the explicit subject of Earman 1985 and Jeffreys 1957. While Earman and Jeffreys tend to talk of universal hypotheses and Popper tends to talk of law-statements nothing here hangs on this difference.

17. In Earman 1985 the notion of being "weakly projectible in the future moving instance sense" is presented under the section heading "Instance Induction: Marching into the Future". In Earman's reprise of this material in his excellent 1992, he repeats (p. 95) the claim that Jeffreys has shown that the assignment of a non-0 prior probability of universal generalizations "is a sufficient condition for instance induction" and there claims that this will be demonstrated later in Section 7 of the same chapter. Yet, interestingly, in that section (pp. 104–113) Jeffreys' result is glossed as "A Proof of the sufficiency for weak Projectibility" and all explicit reference to "instance induction" has been dropped.

$n \geq 1$, $P(\alpha_n) = 1$. Alternatively, he might accept (19) simply because he rejects the Simple Exchangeability Principle $(i)(j)(P(\alpha a_j) = P(\alpha a_i))$. For instance, he might accept the claims, $P(\alpha a_1) = .9$, $P(\alpha a_2) = .99$, $P(P(\alpha a_3) = .999$ etc. In either of these cases his acceptance of (19) involves no inductivist commitment on his part. However, both these deductivist routes to accepting (19) lack credibility. We will now explore a deductivist rationale for accepting (19) which involves neither a rejection of Simple Exchangeability or Regularity.

Suppose a committed deductivist accepts the following prima facie not implausible claims,

$$\text{For any } i, j \varepsilon N, P(a_j = a_i) > 0 \qquad (20)$$

For any $i, j, k, l \varepsilon N$, if $i \neq j$ and $k \neq 1$,

and $$P(a_i = a_j) = P(a_k = a_l), \qquad (21)$$

For any distinct names $a_1, \ldots . a_n$, $P(a_n = a_1 \& \ldots . \& a_n = a_{n-1})$

$$= P(a_n = a_1) \text{ x } \ldots . \text{ x } P(a_n = a_{n-1}). \qquad (22)$$

These together entail

$$\lim_{n \to \infty} P(\alpha a_{n+1} = a_1 \text{ v } \ldots \text{ v } a_{n+1} = a_n) = 1.\text{[18,19]} \qquad (23)$$

Where $P(e') = 1$, $P(h/e) = P(h/e\&e')$. So given (19), (20), and (21),

$$\lim_{n \to \infty} P(\alpha a_{n+1}/(\alpha a_1 \& \ldots \& \alpha a_n))$$
$$= \lim_{n \to \infty} P((\alpha a_{n+1}/(\alpha a_1 \& \ldots \& \alpha a_n)) \qquad (24)$$
$$\& (a_{n+1} = a_1 \text{ v } \ldots \text{ v } a_{n+1} = a_n)) = 1.\text{[20]}$$

Note, the claim

18. Let S be the infinite set consisting of all statements of the form $\ulcorner a_j = a_i \urcorner$ where i, $j \varepsilon N$ and $i \neq j$. Then we could obtain (23) from (22) and assumption that for some infinite sub-set S′ of S, there is some r such that $r > 0$, and for any s, if $s \varepsilon S'$ then $P(s) \geq r$. This assumption is entailed by, but does not entail, the conjunction of (20) and (21).

19. Note, a proof of this can be constructed on the lines of the proof of section 4 above, yielding the result that, where $P(a_1 = a_2) = b$ and $0 < b \leq 1$,

$$\lim_{n \to \infty} P(a_{n+1} = a_1 \text{ v } \ldots \text{ v } a_{n+1} = a_n) = \lim_{n \to \infty} 1 - (1 - b)^n = 1.$$

See the proof of section 4 above and let 'σ_m' abbreviate '$a_{n+1} = a_1$', 'σ_{2m}' abbreviate '$a_{n+1} = a_2$', etc.

20. A referee from this journal claims that (24) does not follow from (20) to (22) unless one adds the premise

$$\text{For any } n, P(\alpha a_1 \& \ldots \& \alpha a_n) \geq e \text{ for some fixed } e > 0. \qquad (22a)$$

As of yet I have not fully understood the referee's grounds for this claim. What I shall note here is that the above argument is framed by the supposition that a deductivist can accept the claim that $P((x)\alpha x) > 0$ without lapsing into inductivism. But note, where

$$\lim_{n\to\infty} P((\alpha a_{n+1}/(\alpha a_1 \ \& \ .. \ \& \ \alpha a_n)) \ \& \ (a_{n+1} = a_1 \vee .. \vee a_{n+1} = a_n)) = 1.$$

does not involve the slightest semblance of inductivism since the evidence statement '$((\alpha a_1 \ \& \ .. \ \& \ \alpha a_n) \ \& \ (a_{n+1} = a_1 \vee .. \vee a_{n+1} = a_n))$' *deductively* entails the hypotheses 'αa_{n+1}'.

The substantive point here is that a deductivist may allow that as positive instances accumulate the probability that the next instance is positive increases. He may do so not because he admits even a whiff of inductivism but because he allows that as instances pile up the probability that the individual mentioned in the next case has already been covered by one of the past cases approaches unity. That is to say, a deductivist can accept (19), not because he induces from the character of known individuals to the character a wholly unknown separate individual, but because as instances pile up he gives an ever greater probability to the claim that the next instance will involve an individual that is already included in one of the previous instances. Paradigmatically, induction involves inferring from *observed individuals* to *unobserved individuals.* This is not the same as inferring from *known instances* to *distinct instances.* Distinct instances need not involve distinct individuals. What makes two instances of a generalization distinct is the fact that they contain orthographically distinct individual constants. This leaves open the possibility that these different constants actually refer to one and the same individual.

Note, it is not being argued here that a deductivist needs to accept any of (20), (21), or (22). The point is that a deductivist who does so may accept Earman's (19) without admitting even a whiff of inductivism. In other words, mere acceptance of (19) does not in itself commit one to some form of inductivism, even where one accepts Regularity and Simple Exchangeability of Instances.[21]

To see how a deductivist can allow that as positive instances of a given universal generalization accumulate the probability that the next instance is positive rises imagine the following scenario. Two bird spot-

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$P((x)\alpha x) > 0$, for any n, $P((x)\alpha x/\alpha a_1 \& \ldots \& \alpha a_n) > 0$. But $P((x)\alpha x/\alpha a_1 \& \ldots \& \alpha a_n) = P((x)\alpha x)/P(\alpha a_1 \& \ldots \& \alpha a_n)$. So where $P((x)\alpha x) > 0$, $P(\alpha a_1 \& \ldots \& \alpha a_n) \geq P((x)\alpha x$, otherwise $P((x)\alpha x)/P(\alpha a_1 \& \ldots \& \alpha a_n) > 1$ contra the axioms of the probability calculus. So $P((x)\alpha x)$ is the required fixed $e > 0$. So if the referee is right we need merely add to (20)–(22) the premise

$$P(x)\alpha x > 0 \qquad (22b)$$

in order to derive the needed (22a). This renovation still preserves our original claim that a deductivist can accept (22b) and hence (19) without lapsing into deductivism.

21. Of course, (19), when combined with certain other claims, for instance, the claims $(i)(j)((i \neq j) \rightarrow P(a_i = a_j) = 0)$, $(i)(j)(P(\alpha a_i) = P(\alpha a_j))$, and $P(\alpha a_1) < 1$, suffices to commit one to some form of inductivism.

ters, Indi and Didi, are trying to determine if all the swans in a particular locale are white. Before making observations both Indi and Didi agree that the probability that any given swan is white is r. After observing many swans all of which are white Indi asks Didi if he agrees that the probability that the next swan they observe is white is greater than r. Didi agrees. With a cry of triumph Indi, a confirmed inductivist, reproaches Didi, a professed deductivist die-hard, "You see Didi you do have inductivist tendencies despite yourself. On the basis of past observed white swans you increase the probability you assign to the claim that the next swan will be white." To this Didi simple replies, "No Indi, you are jumping the gun. While I do allow that our observations of many and only white swans increases the probability that the next swan will be white, I countenance this increase only because the more swans we observe the higher the probability that the next swan we observe is identical to one we have already observed. Indeed, the more swans we observe the more certain I become that the next one we observe will be identical to one we have already observed. Now if we had marked each of the swans we had observed then as we observed more white swans, and assuming there is in fact a next unmarked swan, I would not have increased the probability I attribute to the claim the next unmarked swan will be white. That would be to reason from that which is known to be observed to that which is known to be unobserved. That would be to assign probabilities inductively. In our present case I am merely reasoning from that which is known to be observed to that which has an ever rising probability of being among the already observed. This involves no element of inductivism."[22]

Can the Earman-Jeffreys argument for instance "induction" be fixed up by assuming, à la Popper, that each constant of the language in question names uniquely? Making this assumption would rule out our hypothetical deductivist's assumption (20) which was part of his wholly deductivist argument for Earman's (19). In fact, this kind of attempted fix-up will fail as long as we allow that there may be only finite individuals of the kind in question. To see this consider the following scenario.

Didi and Indi are examining birds for color. Among other things, they are particularly wondering if all swans are white. To make sure they do not count the same swan twice they mark each swan after examining it. After examining and marking many white swans and no non-white swans Indi asks Didi if he agrees that as they examine and mark more and more white swans the probability that for the next thing they examine if it is an unmarked swan then its white has increased.

22. To avoid irrelevant details we here leave aside all considerations about the possibility of swans changing color over time.

Didi agrees. Indi cries with triumph "So Didi, you are an inductivist despite yourself!" To this Didi calmly rejoins "No Indi, I allow no hint of inductivism. However I do allow that there might well be only a finite number of swans in the universe. So as we keep observing distinct white swans and then mark them I raise the probability that for the next thing we examine if it is an unmarked swan then it is white simply because as we examine and mark more and more white swans I continually raise the probability that we have exhausted the population of unmarked swans. Assuming we have exhausted the unmarked swans it follows trivially that if the next thing we examine is an unmarked swan then it will be white."[23]

9. Concluding Remarks. In fact, assuming Instantial Regularity and Simple Exchangeability, something like the Earman-Jeffreys proof may be developed to show that providing we assume there are infinitely many σs we cannot accept that a LUG of the form $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$ has a non-zero prior probability without some commitment to inductivism. This conclusion is an equivalent of our earlier conclusion that, assuming Instantial Regularity and Simple Exchangeability, if we accept that there are infinitely many σs and that instances of LUGS are probabilistically independent of each other, then we are committed to the claim that the LUGs of the form $\ulcorner(x)(\sigma x \rightarrow \phi x)\urcorner$ have a probability of 0.

Popper and Earman agree that if one wishes to avoid inductivist assumptions one must accept the claim that all LUGs have a prior probability of 0. Popper takes this as good grounds for assigning 0 prior probability to all LUGs. Earman takes this as grounds for being an inductivist. This seems to be a classic case of one man's *modus ponens* being another man's *modus tolens:* If some LUGS have a non-zero prior probability then some form of inductivism holds; Popper takes the *modus tolens* option and concludes that all LUGs have a prior probability of 0; Earman suggests we take the *modus ponens* option and conclude that some form of inductivism holds. In fact, we are

23. Note, it is not being claimed that where one takes the population of σs to be finite, increasing the probability of the claim 'If there is a next σ distinct from previously observed σs it will have property φ' on the observation of an σ that has property φ is never inductive. However if one increases the probability merely because that observation leads one to increase the probability that there is no next σ distinct from previously observed σs then one's increase need reflect no inductivism. What makes for inductivism is, for instance, a commitment to raise the probability of 'If there is a next σ distinct from previously observed σs it will have property φ' on the evidence 'Observed σ's have been φ and there will be a next σ distinct from previously observed σs'. Such a commitment is an inductive commitment irrespective of whether one takes the population of σ's to be finite, infinite, or unknown.

not forced to make such a choice. One can accept that some LUGs have a non-zero prior without accepting any inductivist claim. For instance, one might assign a non-zero prior probability to the claim that all ravens are black, while admitting no inductivist thesis, simply because one allows that there might be only a finite numbers of ravens.[24]

In beginning this essay, I sounded a warning about attempts to use the probability calculus to construct merely formal proofs of inductive or inductive sceptical theses. More charitably, one might view Earman, Jeffreys, and perhaps even Popper, as trying to use the probability calculus to show how inductive and inductive sceptical conclusions follow from various other substantive assumptions. In this case I would still urge caution. We need to be cautious, first, about exactly what assumptions are needed to get the arguments going and, second, about exactly which conclusions count as inductive or inductive sceptical conclusions.

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24. The claim that there might be only a finite number of ravens does not conflict with the claim that there are potentially infinitely many ravens. While law-like statements about σ's are true of potentially infinite number of σ's this does not entail that there are in fact infinitely many σ's. If in fact there are only a finite number of electrons in the universe this would not prevent 'All electrons repel each other' being a law statement. A finite world is not ipso facto a lawless world. The conventional requirement that genuine law-like statements be unrestricted does not guarantee infinite numbers of any specific kind of entities.